Positive Train Control with Active Communication System

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Motivation

- The growing demand for capacity calls for more efficient and safer railway infrastructures.
- Advanced technologies could achieve better efficiency while enhance operation safety.
- Communication based train control development.
- The impacts of active communication:
  - Safe headway selection
  - Train dispatching
  - Base station distribution
- **Headway Selection**

- **Train Control without Active Communication**

- Minimum headway: one block length + train length

- Usually much longer than required size
Train Control with Active Communication

- Headway is dynamic and could be optimized
- Worst-case Stopping Scenario
Headway Selection with Active Communication

- $t_{ia}$: Train $i$ starts braking after a response time
- $t_{ib}$: Train $i$ builds full emergency braking rate
- $t_{istop}$: Train $i$ stops
- $t_{ja}$: Train $j$ receives message from Leading Train
- $t_{jd}$: Train $j$ builds full service braking after response of driver and braking system
- $t_{jstop}$: Train $j$ stops

Worst-case Stopping Scenario
Headway Selection

Dynamic Headway Calculation

\[ a_i(t) = a_{k_{\text{adj}}} + \begin{cases} a_i(0) & \text{if } t \leq t_{ia} \\ a_i(0) + \text{jerk}_i \times (t - t_{ia}) & \text{if } t_{ia} < t \leq t_{ib} \\ a_{i_{\text{max}}} & \text{if } t > t_{ib} \end{cases} \]

\[ a_j(t) = a_{k_{\text{adj}}} + \begin{cases} a_j(0) & \text{if } t \leq t_{jc} \\ a_j(0) + \text{jerk}_j \times (t - t_{jc}) & \text{if } t_{jc} < t \leq t_{jd} \\ a_{j_{\text{max}}} & \text{if } t > t_{jd} \end{cases} \]

\[ \min \ x(0) \]
\[ \text{s.t. } x(t) = x(0) + D_i(t) - D_j(t) \geq L_i \]

\[ D_i(t) = \begin{cases} \int_0^t (V_i + \int_0^\tau a_i(\xi) \ d\xi) \ d\tau & \text{if } t \leq t_{istop} \\ D_i(t_{istop}) & \text{if } t > t_{istop} \end{cases} \]

\[ D_j(t) = \begin{cases} \int_0^t (V_j + \int_0^\tau a_j(\xi) \ d\xi) \ d\tau & \text{if } t \leq t_{jstop} \\ D_j(t_{jstop}) & \text{if } t > t_{jstop} \end{cases} \]

\[ H_{i,j,k} = \min \ x(0) + L_{sm} + V_j t_{\text{fail}} \]
Case Study

- **Dispatching with Active Communication**
  - Objective: Minimize the travel time of all trains: \( \min \sum_{i \in T} w_i d_{i,s_i} \)

**Problem Constraints:**
- **Departure Time Constraints:** \( a_{i,s_i} \geq d_i \)
- **Traveling Time and Dwell Time Constraints:**
  \[
  d_{i,k} \geq a_{i,k} + m r_{i,k} + m d_{i,k} \\
  d_{i,s_{i+1}} \geq a_{i,s_{i+1}} + m b_{i,s_{i+1}}
  \]
- **Safe Headway Constraints**
  \[
  \begin{align*}
  a_{j,k} + M x_{i,j,k} & \geq d_{i,k} \\
  a_{i,k} + (1 - x_{i,j,k}) M & \geq d_{j,k}
  \end{align*}
  \]
  \[
  \begin{align*}
  a_{j,k} + M x_{i,j,k} & \geq a_{i,k} + h_{i,j,k} \\
  a_{i,k} + (1 - x_{i,j,k}) M & \geq a_{j,k} + h_{j,i,k}
  \end{align*}
  \]
- \( x_{i,j,k} \in \{0,1\} \) indicates which train pass through track \( k \) first
Case Study

1) 5 passenger trains and 3 freight trains
2) 10 passenger trains and 5 freight trains
3) 15 passenger trains and 10 freight trains
4) 20 passenger trains and 15 freight trains

### TABLE I
**Characteristics of Tested Trains**

<table>
<thead>
<tr>
<th>Train Type</th>
<th>Length (ft)</th>
<th>Max Velocity (ft/min)</th>
<th>Acceleration Rate (ft/min²)</th>
<th>Deceleration Rate (ft/min²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freight</td>
<td>6000</td>
<td>6160</td>
<td>1584</td>
<td>1584</td>
</tr>
<tr>
<td>Passenger</td>
<td>1000</td>
<td>6952</td>
<td>2112</td>
<td>2112</td>
</tr>
</tbody>
</table>

### TABLE II
**Dynamic Headway Policy Testing Results**

<table>
<thead>
<tr>
<th>Case</th>
<th>Average Delay (min)</th>
<th>Average Traveling Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fixed Headway</td>
<td>Dynamic Headway</td>
</tr>
<tr>
<td>1</td>
<td>5.7</td>
<td>2.1</td>
</tr>
<tr>
<td>2</td>
<td>7.5</td>
<td>3.2</td>
</tr>
<tr>
<td>3</td>
<td>15.4</td>
<td>6.8</td>
</tr>
<tr>
<td>4</td>
<td>23.8</td>
<td>10.0</td>
</tr>
</tbody>
</table>
$L = \{l_1, l_2, \ldots, l_m\}$, the set of alternative locations, ft

$y_j = \begin{cases} 1, & \text{if a base station is placed at location } l_j \\ 0, & \text{otherwise} \end{cases}$

c$_j$ = base station placement cost at location $l_j$, dollar

$r_j$ = base station coverage radius at location $l_j$, ft

$z_{j,k} = \begin{cases} 1, & \text{if } l_k \text{ is covered by base station } y_j, \text{i.e. } |l_k - l_j| \leq r_j \\ 0, & \text{otherwise} \end{cases}$

$h_{\text{min}}$ = minimum overlap distance required to finish handover process normally, ft

$h$ = the practical overlap distance in implementation, ft

$J$ = the index set of locations of base stations

![Image showing base station distribution](image)

$$\min \sum_{j=1}^{m} c_j y_j - \kappa \rho = \sum_{j=1}^{m} c_j y_j - \kappa \frac{h-h_{\text{min}}}{h_{\text{min}}}$$

$$\sum_{j=1}^{m} z_{j,k} \geq 1 \quad \rho \geq 0$$

$$|l_q - l_{q+1}| \leq r_q + r_{q+1} - (\rho + 1) h_{\text{min}} \quad l_q, l_{q+1} \in J$$
Define the following variables:

\[ S = \{ S_1, S_2, \ldots, S_n \} \]

\[ x_i = \begin{cases} 1, & \text{if active communication is placed in area } S_i \\ 0, & \text{otherwise} \end{cases} \]

\[ P_i = \text{profit improvement by placing base stations in area } S_i, \text{ which can be predicted by improved track efficiency from the dispatching model} \]

\[ C_i = \text{minimum cost of placing base stations in area } S_i, \text{ value of objective function in model} \]

\[
\max \sum_{i=1}^{n} (P_i - C_i)x_i \\
\text{s.t. } \sum_{i=1}^{n} C_i x_i \leq C \\
\sum_{i=1}^{n} |J_i| x_i \leq J
\]
Conclusion

- Dynamic headway selection in PTC
- Dispatching optimization model considering active communication
- Base Station Distribution: tradeoff between base station cost and operation profit
Thank you