Dispatching Trains through a Double Track Rail System under Exact Travel Time Estimation

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Outline

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- Heuristics
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Motivation

• Railway moves about 40% of freight measured in ton-miles in 2012 (American Association of Railroads)

• The revenue generated by railway freight transportation is $71.6 billion in 2012.

• It is expected that the railway system will experience a 22% increase in the amount of tonnage from 2010 to 2035 (Federal Railroad Administration)

• It is expensive to extend the railway’s infrastructure.

• A better way of managing the railway system is needed.
Motivation

• New technologies bring the potential to improve railway operations.
  - Positive Train Control (PTC) system

• PTC is initially introduced as a system of monitoring and controlling the movement of trains to increase security by reducing human operations.

• It provides each train with the information of trains near it (“locally”) and even trains far away from it (“globally”).

• With the new information provided by PTC system, we can schedule trains in a more efficient way.

• However, to take advantage of PTC system, finer discretization is needed and travel time also needs finer estimation.
Problem Statement

- Given a train at a station adjacent to point G/L/M, what is the best route to a station adjacent to point C/D
- There is a speed limit associated with each track segment and acceleration/deceleration rates are considered.
Modelling Representation

In Lu et al. (2004), the railway track is discretized into different segments which are smallest, indivisible units in this model. Then several segments are grouped into one node to formulate an abstract network.

- There are no junctions within each segment.
- The length of a segment is no more than the maximum train length.
- Each node has the capacity of one.
Challenge

• Travel time over a segment = segment length / speed limit

• Then the problem is just an application of the shortest path problem.

• However...
Each node is marked with its length (feet) and speed limit (feet/min).

Assume deceleration and acceleration rate is 1000 feet/min*min.

Without decel./accel. process:
- Time A→B→D→F = $\frac{10000}{4000} + \frac{10000}{4000} = 5$
- Time A→C→E→F = $\frac{5000}{1000} + \frac{5000}{2000} = 7.5$

Actually, time A→B→D→F = $\frac{4000}{1000} + \frac{(20000-4000\times4000/(2\times1000))}{4000} = 7$
- Time A→C→E→F = 9.25
Example

- What if the node length is 10 times smaller?
- Assume deceleration and acceleration rate is 1000 feet/min*min
- Without decel./accel. process:
  time A->C->E->F = \(\frac{500}{1000} + \frac{500}{2000}\) = 0.75
  time A->B->D->F = 0.5, which is better

Actually, time A->B->D->F = square root(2*2000/1000)=2
  time A->C->E->F = square root(2*1000/1000)=1.414, which is better
Fixed Headway Representation

- In conventional model, a train’s movement is determined by its route and entering velocities at the nodes along its route.
- If given the route, one train’s movement is controlled through signals at the beginning of the nodes.
Fixed Headway Model Representation
Why smaller nodes?

Train A

Train B
Dynamic Headway Representation

Under the help of PTC system
Problem Formulation

The routing problem under consideration can be represented as a Shortest Path Problem in state-dependent network.

Minimize \[ \sum_{(i,j) \in E} f_{ij} \Gamma(v_i, v_j, l_{ij}, \bar{v}_{ij}) \]

Subject to

\[ - \sum_{j:(i,j) \in E} f_{ij} + \sum_{j:(j,i) \in E} f_{ji} = \delta_i \quad \forall i \in V \]  \hspace{1cm} (1)

\[ v_i \leq \sum_{j:(i,j) \in E} f_{ij} \bar{v}_{ij} \quad \forall i \in V \]  \hspace{1cm} (2)

\[ v_i \leq \sum_{j:(j,i) \in E} f_{ji} \bar{v}_{ji} \quad \forall i \in V \]  \hspace{1cm} (3)

\[ v_i \in \Omega_i(f) \quad \forall i \in V \]  \hspace{1cm} (4)

\[ f_{ij} \in \{0,1\} \quad \forall (i,j) \in E \]  \hspace{1cm} (5)
Problem Formulation

From Lu et al. (2004),

- If \( v_{\text{exit}} \leq \bar{v} \), \( v_{\text{enter}} \leq \bar{v} \), \( v_{\text{exit}}^2 - v_{\text{enter}}^2 \leq 2r_a l \), \( v_{\text{enter}}^2 - v_{\text{exit}}^2 \leq 2r_d l \),

\[
\Gamma(v_{\text{enter}}, v_{\text{exit}}, l, \bar{v}) = -\frac{v_{\text{enter}}}{r_a} - \frac{v_{\text{exit}}}{r_d} + \left(\frac{1}{r_a} + \frac{1}{r_d}\right) \sqrt{\frac{r_a v_{\text{exit}}^2 + r_d v_{\text{enter}}^2 + 2r_a r_d l}{r_a + r_d}} \leq \bar{v}
\]

- If \( v_{\text{exit}} \leq \bar{v} \), \( v_{\text{enter}} \leq \bar{v} \), \( v_{\text{exit}}^2 - v_{\text{enter}}^2 \leq 2r_a l \), \( v_{\text{enter}}^2 - v_{\text{exit}}^2 \leq 2r_d l \),

\[
\Gamma(v_{\text{enter}}, v_{\text{exit}}, l, \bar{v}) = \frac{\bar{v} - v_{\text{enter}}}{r_a} + \frac{\bar{v} - v_{\text{exit}}}{r_d} + \frac{1}{\bar{v}} \left( l - \frac{\bar{v}^2 - v_{\text{enter}}^2}{2r_a} - \frac{\bar{v}^2 - v_{\text{exit}}^2}{2r_d} \right) > \bar{v}
\]

- Otherwise

\[
\Gamma(v_{\text{enter}}, v_{\text{exit}}, l, \bar{v}) = +\infty
\]
Double Track Rail System

\( l_{i1}, \bar{v}_{i1} \) are the length and the speed limit for 1\(^{st} \) segment

\( l_{i2}, \bar{v}_{i2} \)

- \( l_{i1}, \bar{v}_{i2} \) are the length and the speed limit for 1\(^{st} \) segment
- This simple infrastructure will make the problem formulation simpler and more concise
Double Track Rail System

Minimize \( \sum_{i=0}^{N-1} \Gamma(v_i, v_{i+1}, l_{i\alpha}, \bar{v}_{i\alpha}) \)

Subject to

\[
\begin{align*}
  l_{i\alpha} &= \alpha_i l_{i1} + (1 - \alpha_i) l_{i2} \quad \forall i \in \{1, \ldots, N\} \\
  \bar{v}_{i\alpha} &= \alpha_i \bar{v}_{i1} + (1 - \alpha_i) \bar{v}_{i2} \quad \forall i \in \{1, \ldots, N\} \\
  v_i &\leq \bar{v}_{i\alpha} \quad \forall i \in \{1, \ldots, N\} \\
  v_{i+1} &\leq \bar{v}_{i\alpha} \quad \forall i \in \{1, \ldots, N\} \\
  \alpha_i &\in \{0, 1\} \quad \forall i \in \{1, \ldots, N\}
\end{align*}
\]
Heuristic Algorithm

• We first try to solve a subproblem with a smaller size through simple enumeration. For example, at step i, we solve the subproblem consisting of segment \(i, i+1, \ldots, i+m\).
• And then all local optimal solutions to the subproblem are put together to give a near-optimal global solution.
## Experimental Results

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<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
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</table>

Relative gap between look ahead heuristic and the optimal solution
Conclusion

• The proposed heuristic method performs well for a double track rail system.
  - Intuitively, it is because the algorithm can adjust its decision dynamically. When a bad decision is made in a previous step, it will most likely be corrected in following steps. In this way the method will avoid being stuck in a local optimal solution.

• However, it may not perform as well when applied to a general rail system. This is one direction for future work.