# A New Approach for Routing Courier Delivery Services with Urgent Demand 

Final Report<br>METRANS Project

July, 2012

Principal Investigator:
Fernando Ordonez
Maged M. Dessouky,
Ph.D.Graduate Student:
Chen Wang

# Daniel J. Epstein Department of Industrial and Systems Engineering University of Southern California <br> Los Angeles, California 



## Disclaimer

The contents of this report reflect the views of the authors, who are responsible for the facts and the accuracy of the information presented herein. This document is disseminated under the sponsorship of the Department of Transportation, University Transportation Centers Program, and California Department of Transportation in the interest of information exchange. The U.S. Government and California Department of Transportation assume no liability for the contents or use thereof. The contents do not necessarily reflect the official views or policies of the State of California or the Department of Transportation. This report does not constitute a standard, specification, or regulation.


#### Abstract

Courier delivery services deal with the problem of routing a fleet of vehicles from a depot to service a set of customers that are geographically dispersed. In many cases, in addition to a regular uncertain demand, the industry is faced with sporadic, tightly constrained, urgent requests. An example of such application is the transportation of medical specimens, where timely, efficient, and accurate delivery is crucial in providing high quality and affordable patient services.

In this work we propose to develop better vehicle routing solutions that can efficiently satisfy random demand over time and rapidly adjust to satisfy these sporadic, tightly constrained, urgent requests. We formulate a multi-trip vehicle routing problem using mixed integer programming. We devise an insertion based heuristic in the first phase, and use stochastic programming with recourse for daily plans to address the uncertainty in customer occurrence. The resource action for daily plans, considers a multi-objective function that maximizes demand coverage, maximizes the quality of delivery service, and minimizes travel cost. Because of the computational difficulty for large size problems, Tabu Search has been used to find an efficient solution to the problem. Simulations have been done on randomly generated data and on a real data set provided by a leading healthcare provider in Southern California. Our approach has shown significant improvement in travel costs as well as in quality of service as measured by route similarity than existing methods.


## Disclosure

The project was funded in entirety under this contract to California Department of Transportation.

## Acknowledgement

We would like to thank METRANS for funding this research.

## Table of Contents

1. Introduction ..... 9
1.1 Background ..... 9
1.2 Problem Description. ..... 10
1.3 Motivation ..... 11
1.4 Structure of the Report ..... 14
2. Literature Review ..... 15
2.1 Healthcare Logistics \& Vehicle Routing Problem ..... 15
2.2 Multi-trip VRP ..... 17
2.3 Stochastic VRP ..... 18
2.4 Customer Service ..... 21
2.5 Research Gap ..... 22
3. Vehicle Routing with Urgent Requests ..... 24
3.1 Model Formulation ..... 24
3.2 A Small Sample Problem ..... 30
4. Heuristic ..... 32
4.1 Insertion ..... 32
4.2 Tabu Search ..... 36
4.3 Master Routes ..... 39
4.4 Daily Plans with Urgent Requests: ..... 40
5. Experimental Results ..... 43
5.1 Data Generation and Input Parameters ..... 43
5.2 Simulations and Results ..... 44
5.4 Simulations and Results with Real-Life Data ..... 52
6. Implementation ..... 54
7. Conclusions ..... 55
References ..... 56
Appendix A. Simulation Results on Randomly Generated Data ..... 64

## List of Tables

Table 3.1: Customer Information for the Sample Problem............................................... 30
Table 5.1: Time Windows of Regular and Urgent Requests
44
Table 5.2: Simulation Results with 50 Customers............................................................. 47
Table 5.3: Simulation Results with 100 Customers.......................................................... 48
Table 5.4: Simulation Results with 500 Customers.......................................................... 49
Table 5.5: Simulation Results with Real-Life Data ........................................................... 53

## List of Figures

Figure 3.1: Customers, Depot, and Lab ..... 25
Figure 3.2: The Optimal Routing Solution for Day 0 / 1 ..... 31
Figure 4.1: Insertion of Customer Request i ..... 35
Figure 4.2: Pickup is followed directly by delivery ..... 36
Figure 4.3: Pickup is followed directly by delivery ..... 36
Figure 4.4: $\lambda$-interchange Operator ..... 38
Figure 4.5: 2-opt Exchange Operator ..... 38
Figure 5.1: City Size and Customer Locations ..... 43
Figure A.1: Travel Time with 50 Customers ..... 64
Figure A.2: Taxi Cost with 50 Customers ..... 65
Figure A.3: Dissimilarity with 50 Customers ..... 66
Figure A.4: Number of Taxi Trips with 50 Customers ..... 67
Figure A.5: Travel Time per Request with 50 Customers ..... 68
Figure A.6: Total Daily Cost with 50 Customers ..... 69
Figure A.7: Travel Time with 100 Customers ..... 70
Figure A.8: Taxi Cost with 100 Customers ..... 71
Figure A. 9 : Dissimilarity with 100 Customers ..... 72
Figure A.10: Number of Taxi Trips with 100 Customers ..... 73
Figure A.11: Travel Time per Request with 100 Customers ..... 74
Figure A.12: Total Daily Cost with 100 Customers ..... 75
Figure A.13: Travel Time with 500 Customers ..... 76
Figure A.14: Taxi Cost with 500 Customers ..... 77
Figure A.15: Dissimilarity with 500 Customers ..... 78
Figure A.16: Number of Taxi Trips with 500 Customers. ..... 79
Figure A.17: Travel Time per Request with 500 Customers ..... 80
Figure A.18: Total Daily Cost with 500 Customers ..... 81

## 1. Introduction

### 1.1 Background

The vehicle routing problem (VRP) is a problem of designing optimal routes of collection or delivery from one or several depots to a number of geographically dispersed customers. This type of problem is faced by many industries such as courier services (i.e. UPS, Federal Express, and Overnight United States Postal Service) and local trucking companies. In recent years, these types of services have experienced tremendous growth. For example, both UPS and Federal Express have a steady increase in annual revenue in the past decade, both exceeding $\$ 30$ billion annually.

These routing applications not only have to schedule efficient routes for uncertain demands, they also have to handle sporadic, tightly constraint, and urgent requests. For example, typical courier services have a deadline (e.g. 5pm) for overnight delivery service. Requests for overnight services that are received after the deadline are not accommodated, although it is possible for these packages to be delivered through some other low cost re-routing process.

Another example application is the transportation of clinical specimens, which is pervasive in the healthcare industry. On a daily basis, millions of specimens are delivered in the United States from dispersed hospitals and clinics to centralized laboratories for testing and reporting. Timely and efficient transportation of specimens is crucial in providing high-quality and affordable patient service in the healthcare industry. The current situation, however, is far from ideal, where lost or delayed delivery of specimen is the most common problem jeopardizing patient safety (Astion et al, 2003). Barenfanger et al. (1999) report that a shorter turnaround time (TAT) of microbiological procedures is correlated with improved clinical outcomes and financial returns. For the cause of excessive TAT, Steindel and Novis (1999) found in their research that specimen transportation problems account for $56.3 \%$ of delays in the collection-to-receipt phase. And according to Steindel and Howanitz (2001) and Holland et al. (2005), the percentage
of excessive laboratory test TAT is significantly correlated with delay in treatment and increased average length of stay in emergency departments. The cost on the transportation of clinical specimens is a significant burden to healthcare systems, especially for urgent cases which require prompt courier services.

There are several unique characteristics in the laboratory courier routing problem that determines the significance of the proposed research. One of them is the nature of the demand. The clinical specimens generally fall into two kinds of delivery time windows in terms of testing and reporting. The urgent ones typically need to be transported within an hour, and regular ones have several hours of turnaround time. Urgent requests occur at random times throughout the day in a laboratory courier service area. In the current practice, many of these urgent requests are delivered by an outsourced courier service, such as taxis. For mid-to-large scale laboratory systems, the cost of handling urgent demands by taxis is significant; therefore, an opportunity of cost reduction is presented by incorporating these urgent demands into the routine vehicle routing systems.

Another characteristic of the laboratory routing system is the two types of the facilities that the testing requests come from, namely hospitals and clinics. Hospitals normally operate around the clock, whereas clinics typically do not require service during nights and weekends. For this reason, optimal routing of courier service will need to take into account the changing demand levels at different time periods. Additionally, the laboratory courier routing problem includes random customer demands in the healthcare industry, which comes from uncertain requests and the strict testing requirements. Also, because most specimens are perishable, the courier must strictly follow the delivery time windows.

### 1.2 Problem Description

We will describe the problem with a real-life example in this section. A leading healthcare provider in the Southern California region operates about 200 medical facilities. The healthcare provider continuously delivers medical samples, lab-specimens, mails, x-rays, and documents etc. between various medical facilities and a central lab for testing. The medical facilities are located throughout Southern California, where the
travel time between facilities is comparable to a complete trip length. This makes this routing problem one in which there are typically few locations visited per trip. The healthcare provider has about 50 vehicles to carry out the deliveries. Because of the random nature of the demand in the healthcare industry, the requests may occur at any time during the day. As most medical samples are perishable and should be processed in a short time period, the demands have time windows for pickup and for delivery. The deterministic routine requests typically have to be delivered to the lab in 4 hours after being collected from the customer; the random urgent requests always have to be delivered to the lab within an hour after being collected. There are no capacity constraints, because the sizes of the samples are small compared to the capacity of the vehicles. The vehicles travel through multiple urban areas in several consecutive trips each day to serve the requests. The vehicle depot is located at the central lab where all routes start and end a trip. Third-party couriers (i.e. taxis) are introduced to serve the unmet demands of the regular fleet.

In this work we propose a multi-trip VRP formulation, with deterministic routine requests and random urgent requests, to represent the real world healthcare routing problem described above. Particularly, we are making a best possible plan for a horizon of several days, with a number of vehicles, to service the customers that send out deterministic and stochastic requests that follow certain time window constraints.

### 1.3 Motivation

Even though there are a number of studies and published results in the routing literature, the scheduling of the urgent request delivery of medical specimens is still a manual process in practice. This basically is because of the nature of the demand. Besides the routine demands, there is a significant amount of urgent demands that occur randomly throughout the day and have tight constraints. The key issue in this problem is how to integrate these uncertain demands into the delivery schedule for the routine demands. This could be achieved by two systems, a regular system for the routine demands and a taxi system for the random urgent ones. However, this is extremely costly, especially for a mid-to-large size system. The second reason for the manual process of specimen
delivery is the continuous nature of the demand. Because of the random nature of customer demands in the healthcare industry, a request for delivery may occur at any time of the day. The third reason for the phenomenon is that most medical specimens are perishable, and therefore must be processed within a short time window. The tight time window requires the algorithms for the routing problem be capable of handling multiple trips.

In this work, we propose to address the gap by developing logistic methods considering these specific requirements of routing clinical specimens. When modeling the specimen delivery system for the healthcare industry, the following aspects must be taken into consideration: healthcare network configuration, nature of delivery requests, the objective of quality of healthcare service, and the cost of unmet demands.

In the current practice, the healthcare delivery service runs fixed daily routes in the planning horizon and services all random requests that cannot be accommodated using taxi. Having fixed daily routes uses routes that are similar each day (exactly the same), and visits customers by the same vehicle at roughly the same time every day. Such stability with routes is desirable in repeating systems where the quality of service is important. If the routes used every day are similar to each other, then drivers become more familiar with the area and it becomes easier to adjust to local changes each day due to the familiarity of the drivers (Groër et al. 2008, Sungur et. al 2010). However, rigid routing strategies, with constant routes, also have a drawback in that it is inefficient when there are plenty of random urgent demands. Currently, most of the random urgent specimen bypass the routing system and use a dedicated vehicle, which introduces a substantial additional cost. The ability to adapt the routing solutions in response to these urgent requests can make a fundamental difference in customer service and operational costs, which is essential in the industry. It should be mentioned that, as an abstract concept, route similarity is a user defined measurement of how a route resembles another. It can correspond to the number of customers that are visited by the same vehicle in different days, or the number of arcs that are repeated (Sungur et. al 2010), or measured in terms of the area that is covered by each vehicle (Zhong et. al 2007). We consider route similarity a benefit, and the other objectives (travel distance and taxi usage) in this study are considered as costs. We create a new measure to represent the degree of
similarity in the routes created. This measure, which we refer to as "dissimilarity" increases by 1 every time a customer is visited by different vehicles in two days.

Simply including the urgent requests as part of the possible demand is not straightforward. If a chance constraint model or a robust optimization approach is used, either the unlikely requests are ignored or the solution considering them is at a high cost. We use an approach called stochastic programming with recourse approach to handle the urgent requests. This approach requires massive number of scenarios, leading to large scale routing problems. To solve this problem, we develop a model with a multi-period time horizon to compare the frequency of urgent requests with that of regular ones. The overall idea is to understand whether we can sacrifice some optimality with regard to regular demand to free some capacity or to obtain more flexible routes, which could accommodate more urgent requests at a lower cost.

In this research, we build a model for this vehicle routing problem, and solve it using heuristic algorithms. The model and the heuristic algorithms take into account the following characteristics of the healthcare delivery application: continuous demand, urgent requests, and multiple objectives. The work is built based on the assumption that it may be possible to satisfy the regular demands in a way that the slack of the vehicles can be used to address urgent requests. We build a multi-trip formulation and use stochastic programming with recourse for the master and daily routes. When formulating the master plan, it is desired that the master plan is similar to the daily plans that have uncertainty in customer occurrence. We use an approach that forms the master plans that would require little modification when adapted to daily schedules. Both the master plan and the recourse action for each daily schedule consider a multi-objective function that minimizes the delivery cost, minimizes taxi usage, and maximizes the quality of the healthcare customer services.

Besides the modeling and the heuristic algorithms, we investigate how different uncertainty modeling decisions impact the quality of the routing solutions. Given the nature of the healthcare delivery problem, we evaluate the quality of the planned master and daily routes under different demand loads in terms of routing efficiency and route similarity. We compare the performance of the routing solutions through simulation under different settings and uncertainty scenarios.

In summary, there are three major contributions we make with this research:

1) Propose a routing model suitable for the healthcare industry courier delivery problem.
2) Develop new heuristic algorithms to solve the problem.
3) Establish recommendations of best practices via simulations.

Even though the stated application is for the healthcare delivery problem, the methods we develop here help the modeling of routing problems with urgent and stochastic demand in general.

### 1.4 Structure of the Report

The rest of the report is organized as follows. In section 2, a literature review of the relevant problems is presented. Section 3 formally introduces the problem formulation. In section 4, various problems in constructing heuristics for solving large size problems are discussed, and a new heuristic for handling a large size problem is proposed. In section 5, experimental results of an application of the proposed heuristic on a hypothetical large-size example are presented and discussed. The future research plan and timetable is presented in section 6.

## 2. Literature Review

In this section, we review the literature relevant to our research. First of all, we give a brief review on the studies in the healthcare logistics and the general vehicle routing problem (VRP). Generally speaking, existing research on healthcare logistics does not consider multi-trip delivery, randomness, and urgency exist in the nature of the demand. To overcome these limitations, the model for the courier delivery problem of medical specimens should take these aspects into consideration. In the following two subsections, we focus on the relevant literature in the class of VRP: multi-trip VRP (MtVRP) and stochastic VRP (SVRP). Next, we briefly review the literature on customer services in the vehicle routing problem. In the last part, we address the gap we fill with this research.

### 2.1 Healthcare Logistics \& Vehicle Routing Problem

Since the most important application of the proposed work is in the logistics and the supply chain systems of the healthcare system, we first review the related papers in healthcare logistics. The research in the logistic and supply chain systems of the healthcare industries has primarily been focused on the pharmaceutical industry (i.e., Papagergiou et al. 2001; Shah 2004; Meller et al. 2009). These models focus on optimizing the inventory systems or medication repackaging options. Other examples studying healthcare system logistics include Nicholson et al. (2004) who study outsourcing inventory management decisions, Jarrett (2006) who investigates the implementation of just-in-time systems; Vissers and Beech (2005) study the management of patient flows between organizations etc.

Another relevant research area for healthcare logistics is home healthcare (HHC) service. HHC is a service that provides nursing assistance to patients, especially to the elderly, in their homes. Usually a HHC service operates a fleet of vehicles that are used to drive the nurses to the patients, where the nurses perform specific jobs. Begur et al (1997) develop an integrated spatial decision support system for scheduling HHC nurses. Both Bertels and Fahle (2006) and Steeg and Schroeder (2007) study the heuristics for home
healthcare problems that is related to the vehicle routing problem and the nurse rostering problem. Ganesh and Narendran (2007) present a multistage heuristic for a vehicle routing problem that involves a single item pickup, delivery under time window constraints; this problem can be applied to blood delivery for a public healthcare system. Hemmelmayr et al. (2009) study the delivery of blood products, analyzing the potential value of switching to a vendor-managed inventory system (VMI); they present solution approaches with integer programming and variable neighborhood search. Bachouch et al. (2009) study the drug delivery problem for homecare, using mixed integer programming.

The courier delivery problem for medical specimens studied by this research falls under the class of the vehicle routing problem (VRP), which was first introduced by Dantzig and Ramser in 1959. Being a fundamental problem in transportation, distribution, and logistics, VRP studies the scheduling a fleet of vehicles to satisfy a set of geographically dispersed demands at minimum cost. General review of the VRP can be found in a number of literatures, such as Toth and Vigo (2002), Fisher (1995), and Laporte and Osman (1995).To the best of our knowledge, there has been no research studying courier delivery in multiple trips with stochastic urgent requests, especially for the healthcare industry.

The vehicle routing problem is known to be NP-hard, because the travelling salesman problem (TSP), a special case of the VRP, is NP-hard. To solve the vehicle routing problem, a number of approaches are proposed in the literature. Exact algorithms (i.e. dynamic programming, branch and bound, branch and cut, branch and price), which can solve the VRP optimally, are only applicable to small-size problems. To solve moderate-size problems, heuristics are proposed and utilized in practice. Heuristics include constructive heuristic (e.g., Clarke and Wright, 1964), two phase heuristics (e.g., Gillett and Miller, 1974), and improvement methods (e.g., Thompson and Psaraftis, 1993). In the past two decades, several metaheuristics (e.g.,Tabu search, genetic algorithms, simulated annealing, neural networks) have been proposed to solve the vehicle routing problems. Gendreau et al. (1994) propose a Tabu search heuristic to solve the vehicle routing problem with route length and capacity restrictions. Baker and Ayechew (2003) develop a genetic algorithm for the basic vehicle routing problem with weight limit and travel distance limit on the vehicles. Breedam (1995) proposes
simulated-annealing based improvement heuristics for the vehicle routing problems. Modares et al. (1999) address several algorithms for the routing problems based on a selforganizing neural network approach. These metaheuristics typically perform a thorough exploration of the solutions, allowing deteriorating even infeasible intermediate ones; some of the metaheuristics maintain a pool of good solutions, which can be recombined to produce even better ones (Ren, 2011).

### 2.2 Multi-trip VRP

Multi-trip VRP (MtVRP), as a variant of the VRP, has gained little attention in the literature. In the MtVRP, vehicles can be used more than once during the planning horizon. Taillard et al. (1996) are the first researcher who studied the problem. They suggest that assigning more routes to a vehicle is a more practical solution in real life. They design an algorithm based on Tabu search, and their algorithm tries to avoid obtaining a local minimum.

The study of Brandao and Mercer (1997) made an improvement to that of Taillard et al. (1996). This article does not only consider multi-trip VRP, using Tabu search, it also includes the delivery time window and the capacity of the vehicles. Moreover, this article assumes the flexible hiring of vehicles. Later, a simplified version of the paper is published by Brandao and Mercer (1998), with comparison of their study to Taillard's algorithm.

Petch and Salhi (2003) integrate the approaches proposed by Taillard et al. (1996) and Brandao and Mercer (1997 \& 1998). Azi et al. (2006) first describe an exact algorithm for solving a multi trip VRP problem of one vehicle with time windows. Salhi and Petch (2007) make a comprehensive literature review on the multi-trip VRP, and present a genetic algorithm based on a heuristic for the solution of MtVRP.

Another variant of the VRP study which considers periodicity of the usage of vehicles is the periodic VRP, which customers have to be visited once or several times in the planning horizon (Angelelli and Speranza, 2002). PVRP extends the classic planning horizon to several days (Hemmelmayr et. al., 2008). Angelelli and Speranza (2002) propose a Tabu search based heuristic for the solution of a PVRP with intermediate
facilities, where vehicles can renew their capacities. Francis and Smilowitz (2006) present a continuous approximation for service choice of a PVRP with capacity constraints. Hemmelmayr et al. (2008) propose a new heuristic for solving PVRP as well as a Periodic Travelling Salesman Problem, based on a neighborhood search. The paper of Alonso et al. (2008) extends the classic VRP to a periodic and multi-trip VRP with site-dependency and proposes a Tabu search based algorithm to solve the problem.

Besides the models with multiple shifts or trips, overtime can also be an important strategy when a multi-trip model is constructed. Overtime has been widely used as an effective option in production planning and scheduling; it is however rarely used in the study of vehicle routing and scheduling problems. Sniezek and Bodin (2002) propose "a Measure of Goodness" criteria for their cost models, which includes capital cost of a vehicle, salary cost of the driver, overtime time, mileage cost, and cost of capacity renewal at the disposal facilities, to solve their Capacitated Arc Routing Problem with Vehicle/Site Dependencies (CARP-VSD). This model confirms that using overtime does help in generating less expensive routes because of the saving in the capital cost of vehicles. In recent years, Zapfel and Bogla (2008) provide a study of a multi-trip vehicle routing and crew scheduling with overtime and outsources options. Ren et al. (2010) introduce the usage of shifts into the VRP, and study a new variant of the VRP, which is with time windows, multi-shifts, and overtime. The results show that the shift dependent heuristics has significant cost savings. However, the proposed Tabu search based algorithm applies only to deterministic cases.

### 2.3 Stochastic VRP

The stochastic VRP (SVRP) introduces uncertainty in the parameters. A general review of the SVRP can be found in Gendreau et al. (1996). The stochastic VRP can be classified based on the following criteria:
(1) Uncertainty in the problem: The uncertainty can be present in several parts of the vehicle routing problem, i.e., the presence of a customer, the level of demand, and the travel and service times. Generally, the related variants of the problem include VRP with
stochastic customers (VRPSC), VRP with stochastic demand (VRPSD), and with stochastic service and travel times (VRPSSTT).
(2) Modeling method: There are several modeling methods prevailing in the literature for solving the SVRP. The most common one is stochastic programming, which can be further divided into chance constraint programming (CCP) and stochastic programming with recourse (SPR).
(3) Solution technique: Similar to the classic VRP, the solution techniques for the SVRP generally fall into two categories: exact methods and heuristic methods.

When the customer demand follows a given probability distribution, the problem is referred to as VRPSD, which consists of routing the vehicles to minimize expected total distance travelled such that all demands are served. The first algorithm for the VRPSD was developed by Tillman (1969) based on a saving's algorithm. Early contributions on the VRPSD also include Stewart and Golden (1983) who apply chance constraint programming and recourse methods in the modeling, and Dror and Trudeau (1986) who illustrate the impact of the direction of a designed route on the expected cost.

When customers are associated with demand that has a probability of being present, the vehicle routing problem becomes VRP with stochastic customers (VRPSC) (also called probabilistic VRP in the literature), which was initially studied by Jezequel (1985) and Jaillet (1988). The routing problems considering both stochastic customers and demands are typically classified as the VRP with stochastic customers and demands (VRPSCD), and it is a combination of the VRPSC and VRPSD.

In the recent literature, VRPSD, VRPSC, and VRPSCD have been studied under two distinct approaches, the "a-priori optimization" approach and the "re-optimization" approach (Secomandi, 2001). Bertsimas (1992) proposes "a-priori sequence" solutions, which define a visiting sequence in advance that includes all the demand and skipping of the nodes or routes which are known to have no demands. Bertsimas and Simchi-Levi (1996) survey the development for the VRPSCD with emphasis on the proposed algorithms. In these variants of the routing problems, a number of models and solutions allow for recourse actions to adjust an "a-priori solution" after the uncertainty is revealed. The recourse actions proposed in the literature include skipping non-occurring customers,
returning to the depot when capacity is exceeded, or complete rescheduling for occurring customers (Jaillet 1988; Bertsimas et al, 1990; Waters 1989).

With respect to the re-optimization approach, routing is dynamic in a sense that it occurs concurrently with service and no a-priori tours are followed (Secomandi, 2001). Dror et al. $(1989,1993)$ propose a Markov decision process for a single-stage and multistage stochastic model to investigate the VRPSD. However, the algorithms for the reoptimization approach are limited in the literature because of the computational difficulty with this approach. Recent papers include Secomandi (2001) and Secomandi and Margot (2009), in which a re-optimization routing policy and a rollout algorithm are developed.

Another class of the SVRP is the VRP with Stochastic Travel Time and Service Time (VRPSSTT), which has received relatively less attention in the VRP literature compared to the VRPSC, VRPSD, and VRPSCD. In the VRPSSTT, the traffic condition on the roads as well as the service time is uncertain. In other words, the travel time between two locations is not a deterministic number, but rather depends on the congestion situation on the roads; the service time for each request is not deterministic, but depending on the vehicle that is performing the service. Kao (1978) first studies the Travelling Salesman Problem with Stochastic Travel Time (TSPSTT) and proposes heuristics based on dynamic programming and implicit enumeration. Carraway et al. (1989) use a generalized dynamic programming methodology to solve the TSPSTT. Laporte et al. (1992) study the VRPSSTT problem and proposes a chance constrained model, a 3-index recourse model, and a two-index recourse model. A branch-and-cut algorithm is proposed for the three models. Besides the above applications on VRP, the VRPSSTT model is also applied to a banking problem and solved with adapting the savings algorithm (Lambert et al. 1993).

Robust optimization, introduced by Ben-Tal and Nemirovski (1998), has also been used in solving vehicle routing problems. Sungur et al. (2008) solve a capacitated VRP problem with uncertain demand on a fixed set of demand nodes. They use the robust optimization technique to formulate a new method for solving the problem, the Robust Vehicle Routing Problem (RVRP). Shen et al. (2009) study a routing problem for minimizing unmet demand with uncertain demand and travel time. They present a chance constraint model and compare it to a robust optimization approach. Sungur et al. (2010)
study a Courier Delivery Problem (CDP), which is a Vehicle Routing Problem with Time Windows Problem (VRPTW) with uncertain service times and customers. After formulating the problem, the authors proposed a two-phase heuristic based on insertion and Tabu search. Robust optimization is used to construct a worst-case service time for the master plan.

### 2.4 Customer Service

The healthcare courier delivery problem differs from the classical vehicle routing problem in a few ways. An important one is that it has a high requirement on the quality of customer service. In the problem we are considering for example, the clinics and hospitals prefer the samples or specimens to be delivered by the same driver in repetitive days. This would not only guarantee the promptness in the processing of the requests, but also warrant the familiarity of the delivery, both of which are key factors for efficient healthcare logistical systems.

Some recent work has included customer service in the models for fixed route delivery systems under stochastic demand (Haughton and Stenger 1998). Haughton (2000) develops a framework for quantifying the benefits of route re-optimization, also under stochastic customer demands. Zhong et al. (2007) propose an efficient way of designing driver service territories, considering uncertainty in customer locations and demand. Their method uses a two stage model: in the strategic level, core service territories are constructed; in the operational level, customers in the non-core territories are assigned on a daily basis to adapt to uncertainty. This approach however does not consider customer time windows. Groer et al. (2008) introduce the Consistent VRP (ConVRP) model. The objective is to obtain routes such that the customers are visited by the same driver at roughly the same time on each day. They develop an algorithm, ConRTR (ConVRP Record-to-Record travel), which first generates a template and then generates daily schedules from the template by skipping non-occurring customers and inserting new customers. Sungur et al. (2010) introduce the concept of "route similarity" as the number of customers of the daily routes that are within a given distance of any customer on the master plan route, and use it as a key measure for developing optimal routing strategies.

### 2.5 Research Gap

Our work is different from the previous research in a few ways. The primary distinction in the domain of multi-trip VRP is that the earlier research on multi-trip VRP has equal length operation period for all vehicles with the routing in one period independent of the next. In this work we allow continuous operation of non-equal length periods for different vehicles. For example, in a planning horizon of one day, an MtVRP may require the customers to be visited twice in two trips in a workday, with the length of a trip fixed. Or a PVRP may have all the customers be visited in one trip each workday during the planning horizon of a week, where the length of a trip of a vehicle is 8 hours per day. In our problem, the vehicles operate in multiple trips each workday during the planning horizon; the planning horizon is usually multiple days; the length of the trips for each vehicle will not be defined at first, but will be flexible according to the time window of the demands. There are multiple trips with varying length during the planning horizon because when we have a vehicle to visit a customer for pickup of a medical specimen, it is required that the specimen should be delivered to the lab by the same vehicle on the same trip. This variant of the VRP that has multiple trips with varying length in continuous operation over multiple days has not yet been studied in the literature.

Another distinction of our work from the previous work is that the latter has mainly focused on developing daily independent routes without considering the integration of regular demand with random urgent requests. This requires the formation of master routes that have the flexibility to integrate a high number of urgent requests that have tight time windows and that may randomly occur any time of the day.

Furthermore, the prior work focuses on an objective of minimizing travel cost, e.g., the total travel distance and vehicle costs. While in the healthcare domain, customer service is another important factor that needs to be taken into consideration. As emphasized by the current practice, similar daily plan is a representation of a high quality of customer service, with which we will have the same driver visiting the same customer such that the promptness and accuracy of delivery is guaranteed. We develop a model
that has both the cost of vehicles and taxis in the objective function, but also includes route similarity as a measure for the quality level of customer service.

## 3. Vehicle Routing with Urgent Requests

### 3.1 Model Formulation

We formulate a multi-trip vehicle routing model for the healthcare industry courier delivery problem, taking into account the efficient scheduling of regular and urgent requests, as well as route similarities. In this section, we provide a mixed integer programming formulation of this multi-trip VRPTW with stochastic clients.

Assume we are making a routing schedule for a healthcare courier delivery service provider. There are $n$ potential customers (hospitals, clinics) in the region that must be visited during a planning horizon by a fleet of identical vehicles. Each day, some hospitals and clinics out of the $n$ potential customers send out a request that patients' specimen should be picked up at the customer location and delivered to the lab, where both the pick-up and the delivery have to follow certain time windows. The locations of all the potential customers are known. However, the information of which customers have requests is only revealed on the day the requests are made.

There is one depot (node 0 ) located at the central lab. Each vehicle should leave the depot at the beginning of the day, and return to the depot at the end of the day. It can also return to the lab anytime during the day when required (i.e., when there are urgent requests that need samples delivered by a certain time at the lab.). As each vehicle has multiple trips, we assume a dummy depot (represented by node $n+1$ ) located also at the central lab to keep track of which trip the request is on. An example can be found in Figure 3.1 where there are five customers. Nodes 1 to 5 are used to represent the customers; node 0 and 6 are used to represent the depot and the dummy one, both of which located at the central lab.

The notation of the model formulation is as follows.

## Figure 3.1: Customers, Depot, and Lab



The routing parameters:
$D$ : set of days in the planning horizon.
$C: \quad$ set of customers, $C=\{1, \ldots, n\}$.
$K$ : set of vehicles.
$W: \quad$ set of daily trips of a vehicle, $W=\{1, \ldots, n\}$.

The cost parameters:
$t_{i j}$ : minimum travel time between node $i$ and $j$.
$\alpha_{t}$ : unit travel cost, dollars per mile.
$\alpha_{o}$ : unit outsource cost, dollars per taxi trip.
$\alpha_{s}$ : unit dissimilarity cost, dollars for each count of dissimilarity.

The stochastic parameters:
$C^{d}$ : set of occurring customer requests on day $d$.
$s_{i}^{d}: \quad$ service time of customer request $i$ on day $d$.
$a_{i}^{d}$ : the earliest time that the customer can be visited for request $i$ on day $d$.
$b_{i}^{d}$ : the latest time that the customer can be visited for request $i$ on day $d$.
$l_{i}^{d}$ : the latest time that the customer request $i$ can be delivered to the lab on day d .

Other parameters:
$M$ : a sufficiently large number.

The routing variables:
$x_{i j k}^{d}= \begin{cases}1, & \text { if vehicle } k \text { travels from node } i \text { to } j \text { on day } d \\ 0, & \text { otherwise }\end{cases}$
$x_{0 i k}^{w d}= \begin{cases}1, & \text { if vehicle } k \text { travels from the depot to cusotmer } i \text { on trip } w \text { on day d } \\ 0, & \text { otherwise }\end{cases}$ $x_{i(n+1) k}^{w d}= \begin{cases}1, & \text { if vehicle } k \text { travels from customer } i \text { to the lab on trip } w \text { on day } d \\ 0, & \text { otherwise }\end{cases}$
$y_{i k}^{d}: \quad$ the time vehicle $k$ arrives at customer $i$ on day $d$.
$y_{0 k}^{w d}$ : the time that vehicle $k$ leaves the depot for its trip $w$ on day $d$.
$y_{(n+1) k}^{w d}: \quad$ the time that vehicle $k$ returns to depot from its trip won day $d$.

The auxiliary demand variables:
$z_{i k}^{w d}= \begin{cases}1, & \text { if vehicle } k \text { visits customer } i \text { on trip } w \text { on day } d \\ 0, & \text { otherwise }\end{cases}$
$u_{i}^{d}= \begin{cases}1, & \text { if customer } i \text { is visited by a taxi on day } d \\ 0, & \text { otherwise }\end{cases}$
$r_{i k}^{d}= \begin{cases}1, & \text { if vehicle } k \text { visits customer } i \text { on either day } d \text { or day } 0, \text { but not both } \\ 0, & \text { otherwise }\end{cases}$

Before we present the mathematical formulation of the model, some clarification on the parameters and decision variables need to be made.

1) The planning horizon has a length of $|\mathrm{D}|$ days; and $\mathrm{d}=0$ is used to represent the planning for the master routes.
2) The maximum number of trips each vehicle can make in a day is $n$. We allow artificial trips that do not deal with any customers, but just "move" from the depot to the lab and back to the depot without spending any actual time.
3) $t_{i j}$ is the minimum travel time between node $i$ and $j$. Particularly, $t_{0 i}$ is the minimum travel distance between the depot and node $i ; t_{i(n+1)}$ is the minimum travel time between node $i$ and the lab.
4) $r_{i k}^{d}$ is defined as the measure of dissimilarity, with mathematical expression $r_{i k}^{d}=\left|\sum_{w \in W} z_{i k}^{w d}-\sum_{w \in W} z_{i k}^{w 0}\right| \cdot r_{i k}^{d}$ equals to 1 if customer $i$ is visited by vehicle $k$ either on day $d$ or on day 0 , but not both. $r_{i k}^{d}$ equals to 0 if customer $i$ is visited by vehicle $k$ both on day $d$ and on day 0 , or on neither days. In other words the dissimilarity is counted as one if a customer is visited by a different vehicle than in the master plan.

Problem formulation:
Minimize

$$
\begin{gathered}
\alpha_{t} \cdot \sum_{d \in D} \sum_{k \in K}\left(\sum_{i \in C^{d}} \sum_{j \in C^{d}, i \neq j} t_{i j} \cdot x_{i j k}^{d}+\sum_{w \in W} \sum_{i \in C^{d}} t_{0 i} \cdot x_{0 i k}^{w d}+\sum_{w \in W} \sum_{i \in C^{d}} t_{i(n+1)} \cdot x_{i(n+1) k}^{w d}\right) \\
+\alpha_{o} \cdot \sum_{d \in D} \sum_{i \in C^{d}} u_{i}^{d}+\alpha_{s} \cdot \sum_{d \in D \backslash\{0\}} \sum_{k \in K} \sum_{i \in C^{d}} r_{i k}^{d}
\end{gathered}
$$

s.t.

Routing constraints:
$\sum_{k \in K} \sum_{j \in C^{d}, j \neq i} x_{j i k}^{d}+\sum_{k \in K} \sum_{w \in W} x_{0 i k}^{w d}+u_{i}^{d}=1, \quad i \in C^{d}, d \in D$
$\sum_{w \in W} x_{i(n+1) k}^{w d}+\sum_{j \in C^{d}, j \neq i} x_{i j k}^{d}=\sum_{w \in W} x_{0 i k}^{w d}+\sum_{j \in C^{d}, j \neq i} x_{j i k}^{d}=\sum_{w \in W} z_{i k}^{w d}, \quad k \in K, i \in C^{d}, d \in D$
$\sum_{i \in C^{d}} x_{0 i k}^{w d}=\sum_{i \in C^{d}} x_{i(n+1) k}^{w d} \leq 1, \quad k \in K, w \in W, d \in D$
$\sum_{i \in C^{d}} x_{0 i k}^{w d} \geq \sum_{i \in C^{d}} x_{i(n+1) k}^{(w+1) d}, \quad k \in K, w \in W, d \in D$
$y_{i k}^{d}+t_{i j}+s_{i}^{d} \leq y_{j k}^{d}+M \cdot\left(1-x_{i j k}^{d}\right), \quad i \in C^{d}, j \in C^{d}, i \neq j, k \in K, d \in D$
$y_{0 k}^{w d}+t_{0 j} \leq y_{j k}^{d}+M \cdot\left(1-x_{0 j k}^{w d}\right)$,
$j \in C^{d}, w \in W, d \in D, k \in K$
$y_{i k}^{d}+t_{i(n+1)}+s_{i}^{d} \leq y_{(n+1) k}^{w d}+M \cdot\left(1-x_{i(n+1) k}^{w d}\right), \quad i \in C^{d}, w \in W, d \in D, k \in K$
$y_{(n+1) k}^{w d} \leq y_{0 k}^{(w+1) d}$,
$w \in W, d \in D, k \in K$
$a_{i}^{d} \leq y_{i k}^{d} \leq b_{i}^{d}$,
$i \in C^{d}, k \in K, d \in D$
$\begin{array}{lr}-M \cdot\left(1-z_{i k}^{w d}\right)+y_{0 k}^{w d} \leq y_{i k}^{d} \leq y_{(n+1) k}^{w d}+M \cdot\left(1-z_{i k}^{w d}\right), & i \in C^{d}, w \in W, d \in D, k \in K \\ y_{(n+1) k}^{w d} \leq l_{i}^{d}+M \cdot\left(1-z_{i k}^{w d}\right), & i \in C^{d}, w \in W, d \in D, k \in K \\ -r_{i k}^{d} \leq \sum_{w \in W} z_{i k}^{w d}-\sum_{w \in W} z_{i k}^{w 0} \leq r_{i k}^{d}, & i \in C^{d}, k \in K, d \in D\end{array}$

Domain constraints:

$$
\begin{array}{lr}
x_{i j k}^{d} \in\{0,1\}, & i \in C^{d}, k \in K, d \in D \\
x_{0 i k}^{w d} \in\{0,1\}, & i \in C^{d}, w \in W, k \in K, d \in D \\
x_{i(n+1) k}^{w d} \in\{0,1\}, & i \in C^{d}, w \in W, k \in K, d \in D \\
y_{i k}^{d} \geq 0, & i \in V^{d}, k \in K, d \in D \\
y_{0 k}^{w d} \geq 0, & w \in W, k \in K, d \in D \\
y_{(n+1) k}^{w d} \geq 0, & w \in W, k \in K, d \in D \\
z_{i k}^{w d} \in\{0,1\}, & i \in C^{d}, k \in K, d \in D \\
r_{i k}^{d} \geq 0, & i \in C^{d}, k \in K, d \in D \\
u_{i}^{d} \in\{0,1\}, & i \in C^{d}, d \in D
\end{array}
$$

As previously described, the healthcare courier delivery problem should focus not only on plans with minimum travelling cost, but also those with high level of customer service. Therefore, the objective function of our model is to minimize a total cost, that is composed of traveling cost, outsourcing cost, and route dissimilarity cost. The travel cost is represented by $\alpha_{t} \cdot \sum_{d \epsilon D} \sum_{k \in K}\left(\sum_{i \in C^{d}} \sum_{j \in C^{d}, i \neq j} t_{i j} \cdot x_{i j k}^{d}+\sum_{w \in W} \sum_{i \in C^{d}} t_{0 i} \cdot x_{0 i k}^{w d}+\right.$ $\left.\sum_{w \in W} \sum_{i \in C^{d}} t_{i(n+1)} \cdot x_{i(n+1) k}^{w d}\right)$, which is the total distance traveled by all the vehicles in the planning horizon. The outsource cost is represented by $\alpha_{o} \cdot \sum_{d \epsilon N_{D}} \sum_{i \epsilon C^{d}} u_{i}^{d}$, which is the total number of trips that a taxi is used to handle the demands unmet by the regular fleet. It should be noted that this term could easily include the total taxi distance if we change it to $\alpha_{o} \cdot \sum_{d \epsilon N_{D}} \sum_{i \epsilon C^{d}} u_{i}^{d} \cdot t_{o i}$. The route dissimilarity is measured by $\alpha_{s} \cdot \sum_{d \epsilon N_{D} \backslash O} \sum_{k \epsilon K} \sum_{i \epsilon C^{d}} r_{i k}^{d}$, the total number of customers in the planning horizon, that are serviced by a vehicle different from the one servicing it in the master plan.

There are two groups of constraints in our model, namely routing constraints and domain constraints. Constraint (1) assures on each day that each customer should be visited directly from the depot, right after a vehicle services customer $j$, or by a taxi when the regular fleet is unavailable. Constraint (2) assures that each vehicle must leave the
customer after visiting it. It also addresses the fact that a customer has to be visited by a vehicle in one of its trips in a day. Constraint (3) ensures that each individual trip should start with leaving the depot and end by returning to the depot. Constraint (4) enforces the usage of early trips as much as possible, which force the empty trips close to the end of the day instead of at the beginning of the day. Constraint (5) assures the relationship of arrival times at customers $i$ and $j$, when customer $j$ is visited right after $i$ is visited.Constraint (6) expresses the relationship of arrival time to customer $j$, when $j$ is the first customer request a vehicle handles in a trip. Constraint (7) expresses the relationship of arrival time to customer $j$, when $j$ is the last customer request a vehicle handles in a trip. Constraint (8) enforces that the finish time of a trip of a vehicle should be no later than the start time of the next trip of the vehicle. Constraint (9) enforces the arrival time of a vehicle at a customer to be in the required time window for handling the customer request. Constraint (10) requires that the arrival time at a customer on a trip should be between the start time and the end time of the trip. Constraint (11) requires that each vehicle should visit the lab before the drop-off deadline of each specimen collected by a vehicle on a trip. Constraint (12) is another representation of our expression for dissimilarity $r_{i k}^{d}=\left|\sum_{w \in W} z_{i k}^{w d}-\sum_{w \in W} z_{i k}^{w 0}\right|$. It removes the usage of the absolute value in the expression, so that the system is linearized. Constraints (13) - (21) are the domain constraints.

The following observation can be made from the model:

1. Multi trip of a vehicle is used because of the time window constraint of the customer requests. In other words, a trip of a vehicle needs to be finished so that all the medical specimens can be delivered to the lab on time.
2. It is optimal to combine some customer requests into one trip of a vehicle, such that the summation of the travelling time of the vehicle is minimized.
3. It is optimal to use the same vehicle to visit the same customers on different days so that the route similarity is increased.
4. When the cost of introducing a third party vehicle (e.g., taxi) is comparatively high, it is more beneficial to use the regular fleet instead of a third party vehicle.

### 3.2 A Small Sample Problem

To illustrate this model, consider the following small example. There is one day (Day 1) in the planning horizon, and another day (Day 0 ) is used to represent the master plan. There are five customers in the small problem. All of them request for delivery service on Day 1, and all of them will be included in the master plan. The location of the customers, the pickup time window, and the deadline for drop-off at the lab are shown in Table 3.1. The vehicle is assumed travelling at a speed of $50 \mathrm{~km} / \mathrm{hour}$. The other coefficients are $\alpha_{\mathrm{t}}=1, \alpha_{\mathrm{s}}=1$, and $\alpha_{o}=10000$. In this small example, we use a high value for taxi cost, to discourage the use of taxis and we can focus on the optimal routes generated with the vehicles by the healthcare provider.

The small instance uses c1 to c5 to represent customer 1 to customer 5 requesting service, and we use c0 and c6 to represent the depot and the lab. Notation d0 and d1 are used to represent the day for the master plan and day 1, and k1 and k2 are used to represent the two vehicles operated by the healthcare provider to handle the service. Notation w1 to w5 are used to represent the five trips that a vehicle can make.

Table 3.1: Customer Information for the Sample Problem

|  | $\mathbf{x}$ | $\mathbf{y}$ | Earliest Pickup | Latest pickup | Latest Drop-off |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Day 1 |  |  |  |  |  |
| Customer 1 | 110 | 70 | 16 | 18 | 22 |
| Customer 2 | 113 | 73 | 13 | 15 | 19 |
| Customer 3 | 113 | 70 | 6 | 8 | 12 |
| Customer 4 | 110 | 73 | 9 | 11 | 15 |
| Customer 5 | 108 | 68 | 9 | 11 | 15 |

The optimal solution can be illustrated by Figure 3.2. Both in the master (day 0) and the daily (day 1) plan, only vehicle k1 is used with two trips. For the first trip, it travels from the depot to customer 3 , then to customer 4 , then customer 5 , and then back to the depot. It travels from the depot to customer 2, then to customer 1, then back to the depot for its second trip. The optimal objective value of the small sample problem is 21.6 km . No taxi is introduced in this problem and there is no cost on route dissimilarity
as the daily plan is the same as the master plan. In this example, artificial trips (trips between node 0 and node 6 ) are observed because in the model we assume the maximum number of trips each vehicle can make is the number of customer requests. In the optimal solution, however, several customer requests can be combined and handled in one trip of a vehicle, making a number of artificial trips left.

Figure 3.2: The Optimal Routing Solution for Day 0 / 1


## 4. Heuristic

As discussed in earlier chapters exact solution methods will only be able to solve small size instances of this problem. As there are $|D|$ days during the planning horizon, and on each vehicle there is $n$ trips (including real trip and artificial trips), then solving a problem with n customers and k vehicles is equivalent to solving a routing problem with $n|D|$ customers with $n|K|$ vehicles. Therefore, heuristic algorithms need to be constructed, in order to solve the problem which is huge in size. In this section, we present a heuristic to solve this courier delivery problem with urgent requests. The heuristic can be divided into four parts. The first part is the insertion algorithm, which is used repeatedly when constructing master and daily routes. The second part is Tabu search, which is used to obtain a near-optimal solution for the routing solutions. Insertion and Tabu search are generic techniques for this problem and are used to obtain efficient routes. The third part is the construction of the master routes, which can be used to build daily routes. The last part is the construction of the daily routes, basically focusing on the handling of the urgent requests. The construction of master routes against the daily routes is a specific separation of the problem in order to be able to manage the problem size.

### 4.1 Insertion

Insertion techniques have been widely used as an efficient method for solving vehicle routing and scheduling problems. Insertion heuristics are popular because they are fast, easy to implement, and produce good solutions, and they are easy to extend to handle complicating constraints. A comprehensive review of insertion heuristics can be found in Campbell et al. (2004).

Our heuristic uses the insertion technique as a basic cell, and builds the master routes and the daily plans by calling the insertion heuristic. The insertion heuristics used for constructing master routes and daily routes are different due to the objective function considered in each. The insertion for constructing master routes needs only to consider the efficiency in travel distance. The insertion for constructing daily routes, however, needs to consider travel distance, as well as cost for taxis and route dissimilarity. One
reason behind this strategy is not to use taxi in the master routes as long as it is feasible to use the fleet of vehicles. The other reason is that route dissimilarity is measured on daily routes against master routes, and it is only meaningful to include dissimilarity in the cost function for inserting requests into daily routes.

```
Algorithm 1: Insertion of request to form master routes
Input: the scheduled routes; a request to insert.
Output: the updated routes or taxi cost.
for all the positions in all the activated routes
find the feasible insertion positions with minimum insertion cost;
if the insertion is feasible then
update the routes;
else if there is a vehicle to activate then
put the request on the new vehicle;
else update the taxi cost;
```

An insertion heuristic for building master routes is introduced (Algorithm 1). On a daily basis, for the customer requests that are not in the master routes, the insertion algorithm changes to Algorithm1.1. In this updated algorithm, the insertion cost is the summation of the travel cost and the dissimilarity cost when inserting the request into the regular fleet; the taxi cost is the summation of the travelling cost and the dissimilarity cost when using the taxi service.

The basic procedure of an insertion is illustrated in Fig 4.1.In Algorithm 1 and Algorithm 1.1, to check the feasibility of an insertion, we need to "update the arrival times" after we tentatively insert a pickup or delivery of a request. The arrival time at each node can be calculated as $A_{i}=\max \left(A_{i-1}, E_{i-1}\right)+t_{i-1, i}$, where node $i-1$ and node $i$ are the two nodes consecutively visited by a vehicle. $A_{i}$ is the arrival time at node $i, E_{i}$ is the earliest time a vehicle can visit node $i$, and $t_{i-1, i}$ is the travel time between node $i-1$ and node $i$.

## Algorithm 1.1: Insertion of a daily request not in the master routes

Input: the scheduled routes; the master routes; a request to insert.
Output: the updated routes.
for all the positions in all routes
find the feasible insertion positions with minimum insertion cost;
calculate taxi cost;
if minimum insertion cost is smaller than taxi cost
then use fleet;
else use taxi;
if use fleet
then update the routes;
if use taxi or infeasible to insert
then update the taxi cost;

In these algorithms, the feasibility of an insertion can be confirmed by checking if $A_{i} \leq L_{i}$ for all the nodes in the route, and $L_{i}$ is the latest time that node $i$ can be visited. It should be noted that each customer request corresponds to the handling of a pickup and a delivery request pair. For the pickup of a customer request, $E_{i}$ is the earliest pickup time of the specimen and $L_{i}$ is the latest pickup time of the specimen. For the delivery of the customer request, $E_{i}$ is set to 0 (the earliest time the vehicle can return to the lab for delivery) and $L_{i}$ is the latest time the specimen has to be delivered at the lab. The insertion of a customer request is feasible, if both the pickup and the delivery of the request are feasible.

## Figure 4.1: Insertion of Customer Request $i$

## Before Insertion:



## After Insertion of $\boldsymbol{d}_{\boldsymbol{i}}$ :



The cost on the distance traveled can be calculated as follows. If the pickup and delivery of a request are inserted as two consecutive nodes, i.e., if the pickup is inserted as node $i-1$ and the delivery is inserted as node $i$ in a route (Figure 4.2), then the insertion cost can be calculate as $t_{i-2, i-1}+t_{i-1, i}+t_{i, i+1}-t_{i-2, i+1}$. If the pickup and delivery of a request are inserted not next to each other, i.e., if the pickup is inserted as node $i-1$ and the delivery is inserted as node $i+a(a \geq 1)$ (Figure 4.3), then the insertion cost can be calculated as $t_{i-2, i-1}+t_{i-1, i}+t_{i+a-1, i+a}+t_{i+a, i+a+1}-t_{i-2, i}-$ $t_{i+a-1, i+a+1}$.

Figure 4.2: Pickup is followed directly by delivery

| Pickup |  |  |  |
| :---: | :---: | :---: | :---: |
| Delivery |  |  |  |
| i-2 | i-1 | i | $\mathrm{i}+1$ |

Figure 4.3: Pickup is followed directly by delivery

| Pickup |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{i}-2$ $\mathrm{i}-1$ i  $\mathrm{i}+\mathrm{a}-1$ <br> $\mathrm{i}+\mathrm{a}$ $\mathrm{i}+\mathrm{a}+1$    |  |  |  |  |  |  |  |  |

The taxi cost is made up of two parts in the algorithms. One is the fixed cost, which is proportion to the number of trips. The other is variable cost, which is in proportion to the distance from the pickup location to the delivery location.

The cost for dissimilarity is calculated by comparing the scheduled routes to the master routes. If a request is serviced by the same vehicle, then the dissimilarity is 0 ; otherwise, it is 1 . It should be noted that we assume the dissimilarity cost is always 1 when a customer is visited by a taxi.

It should be noted that the described insertion heuristic is a parallel insertion procedure. However, to construct the master routes, we first use the activated vehicle for the handling of the delivery requests. We activate a new vehicle when it is not feasible to handle the request with a currently activated vehicle. This approach is favored for less usage of vehicles in the master routes, which is another factor of cost reduction for the healthcare provider. In other words, with this approach, we have a better idea on the minimum number of vehicles we need to maintain to handle the requests.

### 4.2 Tabu Search

As described above, insertion heuristic algorithms are used to build initial solutions for the master and the daily routes. However, in order to obtain efficient solutions, a Tabu search algorithm (Algorithm 2) is developed as the post phase improvement for the master and the daily routes. The implementation of the Tabu search considers the neighborhoods obtained from the standard 2-opt exchange move (Lin, 1965)
and the $\lambda$-interchange move (Osman, 1993). The $\lambda$-interchange operators are generated by randomly selecting two requested from two different routes, and exchanging the requests by interchanging the pickup and the delivery of each request (Figure 4.4). As the problem requires pickup and delivery of a request handled by the same vehicle, it must be assured that the pickup and the delivery of a request stay on the same vehicle. The 2-opt exchange operator is generated by randomly selecting two nodes (pickup or delivery) on a randomly selected vehicle (Figure 4.5). As a package can only be delivered after it is picked up, it must be assured that the delivery of any request is located after the pickup of the request.

```
Algorithm 2: Tabu Search Algorithm
Input: a master plan or a daily plan to improve
Output: improved master plan or daily plan
repeat
randomly chose two routes from the solution generate \(\eta_{\max }\) neighbors from \(\lambda\)-interchange operator
generate \(\gamma_{\text {max }}\) neighbors from 2-opt operator choose the best solution and make the move;
randomly generate tabu tenure \(\theta\) from a uniform distribution \(\mathrm{U}\left(\theta_{\max }, \theta_{\max }\right)\);
if the move is \(\lambda\)-interchange then set the tabu for moving the exchanged requests for \(\theta\) iterations;
else
```

set the tabu for moving the exchanged nodes for $\theta$ iterations;
until no improvement in $I_{\max }$ iterations;
calculate the objective and save the current solution;

In each iteration, the Tabu search generates $\eta_{\max } \lambda$-interchange neighbors and $\gamma_{\max }$ 2-opt neighbors of the current solution. The number of Tabu iterations $\theta$ is a random number uniformly distributed in $\left(\theta_{\min }, \theta_{\max }\right)$. The Tabu search at each iteration moves to the best neighbor. A temporary move to a worse solution is allowed to escape
from the local minimum. The Tabu status is overwritten if the new solution improves from the best solution. The algorithm terminates if there is no improvement in $I_{\max }$ iterations.

The Tabu search algorithm will be applied on both the master routes, and on the daily routes. When it is applied on master routes, the objective is to minimize the total distance traveled, as to have more slack time to accommodate the random requests. When it is applied on daily routes, the objective is to minimize the cost including total distance traveled, and route dissimilarity, as to improve the overall efficiency of the final routes.

Figure 4.4: $\lambda$-interchange Operator
Before the Move:

Route 1

|  | Pickup 1 |  |  | Delivery 1 |
| :--- | :--- | :--- | :--- | :--- |

Route 2 |  |  | Pickup 2 | Delivery 2 |
| :--- | :--- | :--- | :--- | :--- |

After the Move:

Route 1

|  | Pickup 2 |  |  |
| :--- | :--- | :--- | :--- |

Route 2

|  |  | Pickup 1 |  | Delivery 1 |
| :--- | :--- | :--- | :--- | :--- | :--- |

Figure 4.5: 2-opt Exchange Operator
Before the Move:

Route

|  | Pickup 1 | Delivery 1 | Pickup 2 | Delivery 2 |
| :--- | :--- | :--- | :--- | :--- | :--- |

After the Move:

Route

|  | Pickup 1 | Pickup 2 | Delivery 1 $\mid$ Delivery 2 |
| :--- | :--- | :--- | :--- | :--- |

### 4.3 Master Routes

When forming master routes, we need to consider the following conflicting objectives: an efficient template to satisfy regular demands for routine business, but be able to rapidly adapt to random and urgent requests that arises throughout the day. Therefore we face two main challenges when determining the master plan for the courier routing problem:

1. To determine which requests to include in the master plan.
2. How to obtain large scale, multi-shift routing solutions under uncertainty.

The extreme cases in classifying regular and urgent requests are comparatively easy. A customer that requests service every day usually has wide time windows and should be considered a regular request and be included in the master plan. A customer request in a matter of life and death (i.e. testing of compatibility of donor organs) should be considered urgent, and should not be included in the master route as it occurs rarely.

The problem is how to classify routing requests that have wide windows and occur randomly. If the frequency is high, they could be considered regular requests. Requests of this sort should be scheduled in the master routes and skipped in the day they do not appear. If the request occurs rarely then they should not be included in the master plan, but should rather be handled in the most efficient way, such as to be included in some master route or use a separate dedicated vehicle.

We study different configurations of the courier delivery problem to identify how to balance the benefits of master routes and recourse actions to better service urgent requests. Specifically, if the master routes are built to service a large amount of regular requests, most demand points could be satisfied efficiently; however, there will be less slack time left to handle urgent requests which arise later, and will therefore drive up costs. On the other hand, if the master plans are built to service very few regular requests, then it will be less efficient to service most daily requests and determining the daily routes will be more difficult, but more slack could be left in the master routes to handle urgent request. In this situation, it is possible that some vehicles would not be used in the master routes but be used as a dedicated vehicle to handle urgent requests only.

In order to obtain efficient master routes for the courier delivery problem, algorithm 3 is developed as the solution procedure. The idea is to include the customers
that have a high probability of occurrence. An insertion algorithm is used to construct an initial solution for master routes. Tabu search is used to improve the efficiency in travel distance so that more slack is obtained for more random urgent requests. The simulation results on the comparison of the configuration of the master routes can be found in section 5.

## Algorithm 3: Formation of a Master Plan

Input: All the customers to insert; the probability of a customer to request service in a day; a threshold for probability of customer occurring

Output: Master routes
for all the customers do
if the occurring probability of a customer is larger than the threshold then include the customer into the master plan by calling Algorithm 1.1;
end for
improve the master routes with Tabu search by calling Algorithm 2;

### 4.4 Daily Plans with Urgent Requests:

As described earlier, in the first stage, we obtain the solution of an effective master plan, and in the second stage, we adjust the planned routes to handle the urgent requests occurred. The objective of the second stage is to accommodate as many of the urgent requests as possible with the existing resources, including the slack time of the vehicles for the master routes and the dedicated vehicles for urgent requests. In this second stage, we need to quickly modify the master plan to service the updated requests.

An ideal benchmark solution is obtained by solving the problem once the uncertainty is revealed; however, it is impossible to implement because of the size of a real problem and the limitation in the current computational power. Alternative recourse actions to be implemented must have the three objectives:

1. Easy and quick to compute
2. Obtain a high quality solution
3. Easy to execute with small deviations from the planned routes

If the recourse action allows skipping customers then the problem can be approximated by a knapsack problem (Kellerer et al., 2004). The recourse strategy is inspired by the classic recourse strategy (strategy b) in Bertsimas (1992), which assumes the demand will be revealed before the vehicle leaves the depot to service the customer. Therefore, a customer will be skipped if it does not request service in a particular day.

In our strategy, we also make the same assumption that the travel time and the actual demand on each day are known before the vehicle departs from the depot. The recourse action in each day includes skipping the customers in the master routes that do not request service from the master plan and inserting the customers who request service into the existing routes if possible.

The heuristic algorithm for building daily plans by adapting the master plan using recourse action can be found in Algorithm 4. In the next section, we compare our heuristics to other benchmark approaches.

```
Algorithm 4: Formation of Daily Plans
Input: the master plan; daily requests
Output: the daily plans
for each day do
    take the master plan (generated by Algorithm 3) as the initial daily plan;
    for all the requests in the master plan
    if the request does not occur on the day then
    drop the request from the daily plan;
end for
for all the requests on the day do
if a request is NOT included in the master plan then
insert the request into the daily plan by calling algorithm 1.1;
end for
improve the daily plan with Tabu search by calling Algorithm 2;
for all the requests serviced by taxi do
insert the request into the daily plan by calling algorithm 1.1;
end for
```


## 5. Experimental Results

### 5.1 Data Generation and Input Parameters

We test our model and heuristic using simulation on the following randomly generated data set and input parameters. Our vehicles will be servicing the courier delivery demands in a city assumed as a square plane. Consider a city with a twodimensional coordinate system, the boundary of the city is from -10 to 10 miles in both the x -axis and the y -axis. The depot and the only lab where all the vehicles start and end their services every day are located at the center of the city, that is $(0,0)$ on the twodimensional plane. (Fig 5.1)

Figure 5.1: City Size and Customer Locations


The locations of all the potential customers are known a priori, and the potential customers, in each experiment, are uniformly distributed in the city (see Fig 5.1). Some customers request service at a fixed time every day (deterministic requests), while others only request services at a fixed time on some of the days (random requests). Each random request has a probabilitypof occurring on each day where $p$ is sampled from a uniform $[0,1]$ distribution. The earliest pickup time (the earliest time a customer can be visited) of a request is uniformly distributed from 9 am to 5 pm on each day. The latest pickup time (the latest time a customer can be visited) of the request is 30 minutes after the corresponding earliest pickup time. Each request has a latest drop-off time (a deadline by which the sample has to be delivered at the lab); the latest drop-off time for regular requests is 2 hours after its earliest pickup time, and the latest drop-off time for urgent requests is 1 hour after its earliest pickup time (see Table 5.1).

Table 5.1: Time Windows of Regular and Urgent Requests

|  | Earliest Pickup Time <br> (hours) | Latest Pickup Time <br> (hours) | Latest Drop-off Time <br> (hours) |
| :--- | :--- | :--- | :--- |
| Regular Request | [9, 17] Uniformly | $0.5+$ Earliest Pickup Time | $2+$ Earliest Pickup Time |
| Urgent Request | [9, 17] Uniformly | $0.5+$ Earliest Pickup Time | $1+$ Earliest Pickup Time |

We assume a given number of vehicles to service the requests, which might be different in each experiment. And the vehicles drive at an average speed of 30 miles per hour to service the requests.

### 5.2 Simulations and Results

In the section, we show the simulation results with the above assumptions and data inputs. In each experiment, we assume a fixed number of potential requests, a fixed proportion of deterministic requests among all the requests, and a fixed number of available vehicles to handle the requests. The result of each experiment is taken by averaging the results of 10 replications, each of which takes the average result of 10 days. Each replication determines the probability that a customer will request service based on random sampling. In other words, in each replication, we randomly select a number of
requests to be deterministic requests; for the random request, it has probability $p$ of occurrence in each day of the replication. In each day of a replication, we determine the occurrence of each request by sampling based on the probability $p$.

In each experiment, we compare the following four strategies in terms of average travel distance, average taxi cost, average route dissimilarity, average number of taxi trips, average travel distance per requests, and average total cost, on a daily basis.
A. TAXI: schedule all the deterministic requests as master routes using the insertion heuristic algorithm; use a third party courier, i.e., taxi, for all the random requests. (Apply Algorithm 3 with a customer occurrence probability threshold of 1 to build the master routes; handle all the random requests by taxi.)
B. IND: form a schedule independently for each day, using the insertion heuristic. (Use Algorithm 1 to build daily routes independently.)
C. MFIX: schedule the deterministic requests as master routes, and insert the random requests into the scheduled routes on each day. Use taxi if it is infeasible or more expensive to insert the random request into the scheduled routes. (Use Algorithm 3 to build the daily plans with a customer occurrence probability threshold of 1.)
D. MHALF: schedule the deterministic requests and high occurring probability requests (those who have an occurrence probability of 0.5 or higher) as master routes. In the daily schedules, skip the non-occurring customers and insert the unscheduled random requests into the scheduled routes. Use a taxi if it is infeasible or more expensive to insert the random request into the scheduled routes. (Use Algorithm 3 to build the daily plans with a customer occurrence probability threshold of 0.5.)

The parameters we use in the experiments for the Tabu search algorithms are $\eta_{\max }=50, \gamma_{\max }=50, I_{\max }=100, \theta_{\min }=10$, and $\theta_{\max }=10$.

Table 5.2, 5.3 , and 5.4 summarize the simulation results with 50,100 , and 500 customers respectively. Simulations have been done with different combinations on the number of vehicles and cost parameters. In these tables, "\#Customers" gives the number of potential customers; "\#Vehicle" shows the number of vehicles used in the simulation. $\alpha_{t}$ is the unit cost per hour traveled. $\alpha_{o_{-} f}$ is the fixed cost per trip of taxi. $\alpha_{o_{-} v}$ is the varying cost per hour the taxi traveled. $\alpha_{s}$ is the unit cost per count of dissimilarity.

Column "Proportion Fix" shows the proportion of deterministic customers among all the potential customers. Column "Strategy" lists the four strategies we are comparing. Column "Travel" shows the total distance that a vehicle travels per day on average. Column "Taxi Cost" shows the average daily taxi cost. Column "Dissimilarity" shows the average dissimilarity, which is the total number of vehicles used in the daily routes that is different than the one in the master routes. If a taxi is used, then the dissimilarity is increased by one, as we assume that a different taxi will come to service a different request. Moreover, as there is no master routes generated for independent scheduling, the dissimilarity is calculated by comparing the daily routes to the master routes generated in strategy "master fix". Column "\#Taxi Trips" shows the total number of daily taxi trips introduced on average. Column "Travel/Requests" shows the distance that a vehicle travels to service a request on a daily basis on average. Column "Total Cost" shows the average daily total cost including travel cost, taxi cost, and cost on dissimilarity. It is the summation of each type of costs weighted by the unit cost of that type.

The results are also shown in Figures A. 1 - A. 18 in the appendix.

Table 5.2: Simulation Results with 50 Customers

| \#Customers: 50; \#Vehicles: 4; $\alpha \mathrm{t}=1, \alpha_{0} \mathrm{f}=100, \alpha_{0} \mathrm{v}=0.5, \alpha \mathrm{~s}=0.01$; |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Proporti on Fixed | Strategy | Travel | $\begin{array}{r} \text { Taxi } \\ \text { Cost } \\ \hline \end{array}$ | $\begin{aligned} & \text { Dissmila \# } \\ & \text { rity } \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { \# Taxi } \\ & \text { Trips } \\ & \hline \end{aligned}$ | Travel/R equest | $\begin{aligned} & \text { Total } \\ & \text { Cost } \\ & \hline \end{aligned}$ | $\begin{array}{\|l} \text { Proporti } \\ \text { on Fixed } \end{array}$ | Strategy | Travel | $\begin{aligned} & \text { Taxi } \\ & \text { Cost } \\ & \hline \end{aligned}$ | Dissmila rity | $\begin{aligned} & \text { \# Taxi } \\ & \text { Trips } \\ & \hline \end{aligned}$ | Travel/R equest | $\begin{array}{r} \text { Total } \\ \text { Cost } \\ \hline \end{array}$ |
| 0.8 | TAXI | 2.89 | 424.53 | 4.24 | 4.24 | 0.28 | 436.13 | 0.8 | TAXI | 5.61 | 694.87 | 6.94 | 6.94 | 0.3 | 706.15 |
|  | IND | 3.05 | 0 | 21.58 | 0 | 0.27 | 12.43 |  | IND | 6.03 | 294.4 | 17.23 | 2.94 | 0.29 | 306.63 |
|  | MFIX | 3.14 | 0 | 6.08 | 0 | 0.28 | 12.61 |  | MFIX | 6.02 | 300.41 | 7.46 | 3 | 0.29 | 312.52 |
|  | MHALF | 3.05 | 0 | 2.34 | 0 | 0.27 | 12.22 |  | MHALF | 6.12 | 209.28 | 6.58 | 2.09 | 0.29 | 221.58 |
| 0.6 | TAXI | 2.23 | 1012.32 | 10.11 | 10.11 | 0.29 | 1021.34 | 0.6 | TAXI | 4.46 | 1082.41 | 10.81 | 10.81 | 0.3 | 1091.43 |
|  | IND | 2.94 | 0 | 23.32 | 0 | 0.29 | 12.01 |  | IND | 5.79 | 188.27 | 21.69 | 1.88 | 0.3 | 200.06 |
|  | MFIX | 2.93 | 0 | 12.55 | 0 | 0.29 | 11.86 |  | MFIX | 5.72 | 186.29 | 11.76 | 1.86 | 0.3 | 197.84 |
|  | MHALF | 2.98 | 0 | 5.4 | 0 | 0.29 | 11.99 |  | MHALF | 5.85 | 169.25 | 6.8 | 1.69 | 0.3 | 181.01 |
| 0.4 | TAXI | 1.69 | 1391.76 | 13.9 | 13.9 | 0.32 | 1398.66 | 0.4 | TAXI | 3.38 | 1391.76 | 13.9 | 13.9 | 0.32 | 1398.66 |
|  | IND | 2.71 | 0 | 22.84 | 0 | 0.31 | 11.08 |  | IND | 5.35 | 126.18 | 21.65 | 1.26 | 0.32 | 137.1 |
|  | MFIX | 2.73 | 0 | 17.07 | 0 | 0.31 | 11.1 |  | MFIX | 5.24 | 121.19 | 15.73 | 1.21 | 0.31 | 131.82 |
|  | MHALF | 2.72 | 0 | 7.8 | 0 | 0.31 | 10.95 |  | MHALF | 5.35 | 153.21 | 8.23 | 1.53 | 0.32 | 163.99 |
| 0.2 | TAXI | 1.01 | 2005.57 | 20.03 | 20.03 | 0.38 | 2009.81 | 0.2 | TAXI | 2.04 | 2005.57 | 20.03 | 20.03 | 0.38 | 2009.84 |
|  | IND | 2.62 | 0 | 23.94 | 0 | 0.34 | 10.73 |  | IND | 5.15 | 54.08 | 23.58 | 0.54 | 0.34 | 64.61 |
|  | MFIX | 2.61 | 0 | 22.5 | 0 | 0.34 | 10.65 |  | MFIX | 5.09 | 68.1 | 21.83 | 0.68 | 0.34 | 78.51 |
|  | MHALF | 2.59 | 0 | 8.13 | 0 | 0.34 | 10.44 |  | MHALF | 5.15 | 60.09 | 7.53 | 0.6 | 0.34 | 70.46 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Proporti on Fixed | Strategy | Travel | $\qquad$ <br> Tax <br> Cost | Dissmila \# rity | \# Taxi <br> Trips | Travel/R equest | $\begin{aligned} & \text { Total } \\ & \text { Cost } \\ & \hline \end{aligned}$ | $\begin{array}{\|l\|} \hline \text { Proporti } \\ \text { on Fixed } \end{array}$ | Strategy | Travel | $\begin{array}{l\|l} \hline \text { Taxi } & \text { I } \\ \text { Cost } & \text { r } \\ \hline \end{array}$ | Dissmila rity | \# Taxi <br> Trips | Travel/R equest | $\begin{array}{r} \text { Total } \\ \text { Cost } \\ \hline \end{array}$ |
| 0.8 | TAXI | 2.88 | 424.53 | 4.24 | 4.24 | 0.28 | 860.06 | 0.8 | TAXI | 5.61 | 694.87 | 6.94 | 6.94 | 0.3 | 1400.09 |
|  | IND | 3.09 | 0 | 24.17 | 0 | 0.28 | 2429.37 |  | IND | 5.98 | 222.31 | 21.22 | 2.22 | 0.28 | 2356.27 |
|  | MFIX | 3.2 | 0 | 4.24 | 0 | 0.28 | 436.79 |  | MFIX | 6.01 | 324.45 | 6.94 | 3.24 | 0.29 | 1030.48 |
|  | MHALF | 3.11 | 0 | 1.18 | 0 | 0.28 | 130.44 |  | MHALF | 6.22 | 230.33 | 5.34 | 2.3 | 0.29 | 776.77 |
| 0.6 | TAXI | 2.22 | 1012.32 | 10.11 | 10.11 | 0.29 | 2032.19 | 0.6 | TAXI | 4.48 | 1082.41 | 10.81 | 10.81 | 0.3 | 2172.38 |
|  | IND | 2.97 | 0 | 24.22 | 0 | 0.29 | 2433.89 |  | IND | 5.78 | 155.23 | 21.72 | 1.55 | 0.3 | 2338.78 |
|  | MFIX | 3.09 | 0 | 10.11 | 0 | 0.31 | 1023.37 |  | MFIX | 5.77 | 225.34 | 10.81 | 2.25 | 0.3 | 1317.87 |
|  | MHALF | 3.09 | 0 | 2.38 | 0 | 0.31 | 250.37 |  | MHALF | 5.94 | 191.28 | 4.75 | 1.91 | 0.31 | 678.15 |
| 0.4 | TAXI | 1.67 | 1391.76 | 13.9 | 13.9 | 0.32 | 2788.44 | 0.4 | TAXI | 3.37 | 1391.76 | 13.9 | 13.9 | 0.32 | 2788.49 |
|  | IND | 2.72 | 0 | 23.73 | 0 | 0.31 | 2383.89 |  | IND | 5.35 | 124.18 | 21.66 | 1.24 | 0.32 | 2300.89 |
|  | MFIX | 2.87 | 0 | 13.9 | 0 | 0.33 | 1401.5 |  | MFIX | 5.32 | 147.23 | 13.9 | 1.47 | 0.32 | 1547.87 |
|  | MHALF | 2.84 | 0 | 4.01 | 0 | 0.32 | 412.35 |  | MHALF | 5.42 | 165.24 | 5.72 | 1.65 | 0.32 | 748.07 |
| 0.2 | TAXI | 1.02 | 2005.57 | 20.03 | 20.03 | 0.38 | 4012.64 | 0.2 | TAXI | 2.02 | 2005.57 | 20.03 | 20.03 | 0.38 | 4012.62 |
|  | IND | 2.6 | 0 | 24.38 | 0 | 0.34 | 2448.4 |  | IND | 5.1 | 69.1 | 24 | 0.69 | 0.34 | 2479.31 |
|  | MFIX | 2.73 | 0 | 20.03 | 0 | 0.36 | 2013.93 |  | MFIX | 5.29 | 92.14 | 20.03 | 0.92 | 0.36 | 2105.71 |
|  | MHALF | 2.68 | 0 | 5.04 | 0 | 0.35 | 514.7 |  | MHALF | 5.21 | 66.1 | 5.49 | 0.66 | 0.35 | 625.52 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Proporti on Fixed | Strategy | Travel | $\qquad$ <br> Tax <br> Cost | Dissmila \# rity | \# Taxi <br> Trips | Travel/R equest | $\begin{aligned} & \text { Total } \\ & \text { Cost } \\ & \hline \end{aligned}$ | $\begin{array}{\|l} \text { Proporti } \\ \text { on Fixed } \end{array}$ | Strategy | Travel | $\begin{array}{l\|l} \text { Taxi } & \text { I } \\ \text { Cost } & \text { r } \\ \hline \end{array}$ | Dissmila rity | \# Taxi <br> Trips | Travel/R equest | $\begin{aligned} & \text { Total } \\ & \text { Cost } \\ & \hline \end{aligned}$ |
| 0.8 | TAXI | 2.91 | 2.65 | 4.24 | 4.24 | 0.29 | 438.28 | 0.8 | TAXI | 5.63 | 4.34 | 6.94 | 6.94 | 0.3 | 709.61 |
|  | IND | 2.84 | 1.43 | 24.37 | 2.08 | 0.27 | 2449.79 |  | IND | 5.37 | 2.74 | 23.02 | 4.05 | 0.26 | 2315.49 |
|  | MFIX | 3.13 | 0.36 | 4.24 | 0.52 | 0.28 | 436.9 |  | MFIX | 5.89 | 2.17 | 6.94 | 3.43 | 0.28 | 707.96 |
|  | MHALF | 3.07 | 0.07 | 1.18 | 0.1 | 0.27 | 130.35 |  | MHALF | 6.13 | 1.84 | 5.34 | 2.91 | 0.29 | 548.09 |
| 0.6 | TAXI | 2.22 | 6.37 | 10.11 | 10.11 | 0.29 | 1026.24 | 0.6 | TAXI | 4.41 | 6.81 | 10.81 | 10.81 | 0.3 | 1096.63 |
|  | IND | 2.6 | 1.99 | 24.83 | 2.89 | 0.28 | 2495.39 |  | IND | 5.03 | 2.82 | 22.77 | 4.14 | 0.28 | 2289.87 |
|  | MFIX | 2.85 | 0.91 | 10.11 | 1.31 | 0.29 | 1023.33 |  | MFIX | 5.41 | 2.13 | 10.81 | 3.23 | 0.29 | 1093.95 |
|  | MHALF | 2.99 | 0.25 | 2.38 | 0.37 | 0.3 | 250.22 |  | MHALF | 5.88 | 1.33 | 4.75 | 2.06 | 0.31 | 488.09 |
| 0.4 | TAXI | 1.7 | 8.71 | 13.9 | 13.9 | 0.32 | 1405.49 | 0.4 | TAXI | 3.41 | 8.71 | 13.9 | 13.9 | 0.32 | 1405.54 |
|  | IND | 2.34 | 1.99 | 23.51 | 2.9 | 0.29 | 2362.34 |  | IND | 4.47 | 2.7 | 22.15 | 3.95 | 0.29 | 2226.65 |
|  | MFIX | 2.57 | 1.22 | 13.9 | 1.77 | 0.31 | 1401.5 |  | MFIX | 4.92 | 1.89 | 13.9 | 2.81 | 0.3 | 1401.74 |
|  | MHALF | 2.78 | 0.26 | 4.01 | 0.38 | 0.32 | 412.4 |  | MHALF | 5.33 | 1.27 | 5.72 | 1.95 | 0.32 | 583.92 |
| 0.2 | TAXI | 1.01 | 12.58 | 20.03 | 20.03 | 0.38 | 2019.62 | 0.2 | TAXI | 2.02 | 12.58 | 20.03 | 20.03 | 0.38 | 2019.62 |
|  | IND | 2.13 | 2.18 | 24.55 | 3.16 | 0.31 | 2465.69 |  | IND | 4.21 | 2.34 | 24.19 | 3.41 | 0.31 | 2429.76 |
|  | MFIX | 2.31 | 1.81 | 20.03 | 2.62 | 0.33 | 2014.05 |  | MFIX | 4.5 | 2.17 | 20.03 | 3.19 | 0.33 | 2014.18 |
|  | MHALF | 2.6 | 0.34 | 5.04 | 0.49 | 0.35 | 514.75 |  | MHALF | 5.04 | 0.73 | 5.49 | 1.09 | 0.34 | 559.81 |

Table 5.3: Simulation Results with100 Customers

|  |  |  |  |  |  |  |  | \#Customers: 100; \#Vehicles: 4; $\alpha \mathrm{t}=1, \alpha_{0} \mathrm{f}=100$, $\alpha 0 \_\mathrm{v}=0.5$, $\alpha \mathrm{s}=0.01$; |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Proporti on Fixed | Strategy | Travel | $\begin{aligned} & \text { Taxi } \\ & \text { Cost } \end{aligned}$ | Dissmila rity | \# Taxi <br> Trips | Travel/R equest | Total Cost | $\begin{array}{\|l} \text { Proporti } \\ \text { on Fixed } \end{array}$ | Strategy | Travel | $\begin{aligned} & \text { Taxi } \\ & \text { Cost } \end{aligned}$ | Dissmila rity | \# Taxi Trips | Travel/R equest | Total Cost |
| 0.8 | TAXI | 2.31 | 969.30 | 9.68 | 9.68 | 0.23 | 987.90 | 0.8 | TAXI | 4.64 | 999.34 | 9.98 | 9.98 | 0.23 | 1017.99 |
|  | IND | 2.61 | 0.00 | 55.72 | 0.00 | 0.23 | 21.46 |  | IND | 5.25 | 45.07 | 54.09 | 0.45 | 0.23 | 66.61 |
|  | MFIX | 2.63 | 0.00 | 13.84 | 0.00 | 0.23 | 21.17 |  | MFIX | 5.23 | 104.17 | 13.74 | 1.04 | 0.23 | 125.21 |
|  | MHALF | 2.63 | 0.00 | 5.57 | 0.00 | 0.23 | 21.06 |  | MHALF | 5.20 | 64.09 | 6.60 | 0.64 | 0.23 | 84.97 |
| 0.6 | TAXI | 1.91 | 2060.73 | 20.58 | 20.58 | 0.25 | 2076.23 | 0.6 | TAXI | 3.88 | 2070.74 | 20.68 | 20.68 | 0.25 | 2086.45 |
|  | IND | 2.54 | 0.00 | 55.64 | 0.00 | 0.25 | 20.90 |  | IND | 5.11 | 42.07 | 54.46 | 0.42 | 0.25 | 63.07 |
|  | MFIX | 2.54 | 0.00 | 26.60 | 0.00 | 0.25 | 20.58 |  | MFIX | 5.11 | 49.09 | 25.92 | 0.49 | 0.25 | 69.77 |
|  | MHALF | 2.50 | 0.00 | 8.92 | 0.00 | 0.24 | 20.06 |  | MHALF | 4.98 | 39.07 | 9.94 | 0.39 | 0.25 | 59.08 |
| 0.4 | TAXI | 1.45 | 2844.79 | 28.41 | 28.41 | 0.27 | 2856.64 | 0.4 | TAXI | 2.89 | 2844.79 | 28.41 | 28.41 | 0.27 | 2856.64 |
|  | IND | 2.38 | 0.00 | 51.23 | 0.00 | 0.27 | 19.58 |  | IND | 4.75 | 20.03 | 51.38 | 0.20 | 0.27 | 39.56 |
|  | MFIX | 2.39 | 0.00 | 33.86 | 0.00 | 0.27 | 19.47 |  | MFIX | 4.70 | 28.05 | 33.82 | 0.28 | 0.26 | 47.20 |
|  | MHALF | 2.38 | 0.00 | 11.32 | 0.00 | 0.27 | 19.13 |  | MHALF | 4.68 | 23.04 | 12.52 | 0.23 | 0.26 | 41.90 |
| 0.2 | TAXI | 0.94 | 3765.88 | 37.61 | 37.61 | 0.33 | 3773.76 | 0.2 | TAXI | 1.87 | 3765.88 | 37.61 | 37.61 | 0.33 | 3773.75 |
|  | IND | 2.21 | 0.00 | 49.71 | 0.00 | 0.29 | 18.19 |  | IND | 4.41 | 12.02 | 49.94 | 0.12 | 0.29 | 30.14 |
|  | MFIX | 2.22 | 0.00 | 42.08 | 0.00 | 0.29 | 18.17 |  | MFIX | 4.38 | 22.04 | 41.62 | 0.22 | 0.29 | 39.99 |
|  | MHALF | 2.17 | 0.00 | 14.84 | 0.00 | 0.29 | 17.48 |  | MHALF | 4.34 | 12.02 | 15.18 | 0.12 | 0.29 | 29.52 |
| \#Custome |  | \#Vehic <br> Travel | ; $\boldsymbol{\alpha} \mathbf{t}=$ | $\mathrm{f}=10$ | $\mathrm{v}=0$ | 10 |  | \#Custome |  | \#Vehicles: 4 ; $\boldsymbol{\alpha} \mathbf{t}=1$, <br> Travel <br> Taxi |  | do_f=100, | , $\alpha 0 . \mathrm{v}=0.5$ | , $\alpha s=100$; |  |
| Proporti on Fixed | Strategy |  | $\begin{aligned} & \text { Taxi } \\ & \text { Cost } \end{aligned}$ | Dissmila rity | \# Taxi <br> Trips | Travel/R equest | Total Cost | $\begin{array}{\|l} \text { Proporti } \\ \text { on Fixed } \end{array}$ | Strategy |  |  | Dissmila rity | \# Taxi Trips | Travel/R equest | $\begin{aligned} & \text { Total } \\ & \text { Cost } \\ & \hline \end{aligned}$ |
| 0.8 | TAXI | 2.33 | 969.30 | 9.68 | 9.68 | 0.23 | 1955.90 | 0.8 | TAXI | 4.70 | 999.34 | 9.98 | 9.98 | 0.23 | 2016.13 |
|  | IND | 2.65 | 0.00 | 58.21 | 0.00 | 0.23 | 5842.18 |  | IND | 5.29 | 53.08 | 55.02 | 0.53 | 0.23 | 5576.24 |
|  | MFIX | 2.69 | 0.00 | 9.68 | 0.00 | 0.24 | 989.48 |  | MFIX | 5.30 | 116.21 | 9.98 | 1.16 | 0.24 | 1135.41 |
|  | MHALF | 2.71 | 0.00 | 2.35 | 0.00 | 0.24 | 256.66 |  | MHALF | 5.29 | 95.14 | 4.06 | 0.95 | 0.24 | 522.31 |
| 0.6 | TAXI | 1.93 | 2060.73 | 20.58 | 20.58 | 0.25 | 4134.14 | 0.6 | TAXI | 3.77 | 2070.74 | 20.68 | 20.68 | 0.25 | 4153.84 |
|  | IND | 2.55 | 0.00 | 55.80 | 0.00 | 0.25 | 5600.41 |  | IND | 5.08 | 49.08 | 54.13 | 0.49 | 0.25 | 5482.38 |
|  | MFIX | 2.69 | 0.00 | 20.58 | 0.00 | 0.26 | 2079.51 |  | MFIX | 5.34 | 49.09 | 20.68 | 0.49 | 0.26 | 2138.43 |
|  | MHALF | 2.59 | 0.00 | 4.50 | 0.00 | 0.25 | 470.72 |  | MHALF | 5.06 | 75.13 | 5.70 | 0.75 | 0.25 | 665.36 |
| 0.4 | TAXI | 1.44 | 2844.79 | 28.41 | 28.41 | 0.27 | 5697.30 | 0.4 | TAXI | 2.93 | 2844.79 | 28.41 | 28.41 | 0.27 | 5697.51 |
|  | IND | 2.38 | 0.00 | 52.34 | 0.00 | 0.27 | 5253.03 |  | IND | 4.71 | 12.02 | 51.40 | 0.12 | 0.26 | 5170.85 |
|  | MFIX | 2.54 | 0.00 | 28.41 | 0.00 | 0.28 | 2861.33 |  | MFIX | 4.99 | 40.07 | 28.41 | 0.40 | 0.28 | 2901.03 |
|  | MHALF | 2.49 | 0.00 | 7.36 | 0.00 | 0.28 | 755.89 |  | MHALF | 4.88 | 26.04 | 8.08 | 0.26 | 0.27 | 853.57 |
| 0.2 | TAXI | 0.93 | 3765.88 | 37.61 | 37.61 | 0.32 | 7534.31 | 0.2 | TAXI | 1.86 | 3765.88 | 37.61 | 37.61 | 0.32 | 7534.34 |
|  | IND | 2.21 | 0.00 | 50.30 | 0.00 | 0.29 | 5047.70 |  | IND | 4.41 | 13.02 | 49.81 | 0.13 | 0.29 | 5011.65 |
|  | MFIX | 2.36 | 0.00 | 37.61 | 0.00 | 0.31 | 3779.86 |  | MFIX | 4.69 | 15.02 | 37.61 | 0.15 | 0.31 | 3794.77 |
|  | MHALF | 2.31 | 0.00 | 10.10 | 0.00 | 0.30 | 1028.46 |  | MHALF | 4.60 | 14.03 | 10.20 | 0.14 | 0.30 | 1052.44 |
| \#Customers: 100; |  | \#Vehicle <br> Travel | 8; $\alpha$ t= | $0 . \mathrm{f}=0.5$, | $\boldsymbol{\alpha 0}$ _v=0 | as $=100$ |  | \#Customers: 100; |  | \#Vehic <br> Travel | 4; $\alpha \mathbf{t}=$ | $\boldsymbol{\alpha 0}$ ¢ $\mathrm{f}=0.5$, | $\boldsymbol{\alpha o}{ }^{\text {d }}$ =0. | $\alpha \mathrm{s}=100$; |  |
| Proporti on Fixed | Strategy |  | $\begin{aligned} & \text { Taxi } \\ & \text { Cost } \end{aligned}$ | Dissmila rity | $\begin{aligned} & \text { \# Taxi } \\ & \text { Trips } \\ & \hline \end{aligned}$ | Travel/R equest | Total <br> Cost | $\begin{array}{\|l} \text { Proporti } \\ \text { on Fixed } \end{array}$ | Strategy |  | Taxi <br> Cost | Dissmila rity | $\begin{aligned} & \text { \# Taxi } \\ & \text { Trips } \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { Travel/R } \\ & \text { equest } \end{aligned}$ | $\begin{aligned} & \text { Total } \\ & \text { Cost } \\ & \hline \end{aligned}$ |
| 0.8 | TAXI | 2.35 | 6.14 | 9.68 | 9.68 | 0.23 | 992.92 | 0.8 | TAXI | 4.70 | 6.33 | 9.98 | 9.98 | 0.23 | 1023.15 |
|  | IND | 2.44 | 3.23 | 56.29 | 4.68 | 0.23 | 5651.76 |  | IND | 4.88 | 3.57 | 55.51 | 5.19 | 0.23 | 5574.11 |
|  | MFIX | 2.57 | 0.83 | 9.68 | 1.21 | 0.23 | 989.41 |  | MFIX | 5.13 | 1.22 | 9.98 | 1.81 | 0.23 | 1019.73 |
|  | MHALF | 2.72 | 0.13 | 2.35 | 0.19 | 0.24 | 256.90 |  | MHALF | 5.31 | 0.73 | 4.06 | 1.13 | 0.24 | 427.95 |
| 0.6 | TAXI | 1.89 | 13.02 | 20.58 | 20.58 | 0.25 | 2086.16 | 0.6 | TAXI | 3.77 | 13.08 | 20.68 | 20.68 | 0.25 | 2096.15 |
|  | IND | 2.31 | 3.51 | 55.44 | 5.08 | 0.24 | 5566.03 |  | IND | 4.63 | 3.75 | 54.28 | 5.44 | 0.24 | 5450.26 |
|  | MFIX | 2.57 | 1.15 | 20.58 | 1.66 | 0.26 | 2079.67 |  | MFIX | 5.02 | 1.58 | 20.68 | 2.31 | 0.25 | 2089.68 |
|  | MHALF | 2.53 | 0.20 | 4.50 | 0.29 | 0.25 | 470.48 |  | MHALF | 5.10 | 0.56 | 5.70 | 0.84 | 0.25 | 590.95 |
| 0.4 | TAXI | 1.44 | 18.00 | 28.41 | 28.41 | 0.27 | 2870.55 | 0.4 | TAXI | 2.88 | 18.00 | 28.41 | 28.41 | 0.27 | 2870.52 |
|  | IND | 2.12 | 3.32 | 52.04 | 4.82 | 0.25 | 5224.27 |  | IND | 4.24 | 3.34 | 51.71 | 4.85 | 0.25 | 5191.29 |
|  | MFIX | 2.30 | 1.85 | 28.41 | 2.69 | 0.27 | 2861.27 |  | MFIX | 4.63 | 2.01 | 28.41 | 2.92 | 0.27 | 2861.53 |
|  | MHALF | 2.38 | 0.51 | 7.36 | 0.74 | 0.27 | 755.58 |  | MHALF | 4.63 | 0.79 | 8.08 | 1.16 | 0.26 | 827.31 |
| 0.2 | TAXI | 0.91 | 23.68 | 37.61 | 37.61 | 0.32 | 3791.96 | 0.2 | TAXI | 1.85 | 23.68 | 37.61 | 37.61 | 0.32 | 3792.10 |
|  | IND | 1.94 | 3.12 | 49.73 | 4.52 | 0.28 | 4991.61 |  | IND | 3.85 | 3.17 | 49.74 | 4.60 | 0.28 | 4992.58 |
|  | MFIX | 2.07 | 2.57 | 37.61 | 3.73 | 0.29 | 3780.16 |  | MFIX | 4.13 | 2.65 | 37.61 | 3.84 | 0.29 | 3780.16 |
|  | MHALF | 2.23 | 0.59 | 10.10 | 0.85 | 0.30 | 1028.39 |  | MHALF | 4.46 | 0.71 | 10.20 | 1.02 | 0.30 | 1038.56 |

Table 5.4: Simulation Results with 500 Customers

| \#Customers: 500; |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Proporti on Fixed | Strategy | Travel | Taxi <br> Cost | Dissmila rity | $\begin{aligned} & \text { \# Taxi } \\ & \text { Trips } \\ & \hline \end{aligned}$ | Travel/R equest | Total <br> Cost | Proporti on Fixed | Strategy | Travel | Taxi <br> Cost | Dissmila rity | \# Taxi <br> Trips | Travel/R equest | Total <br> Cost |
| 0.8 | TAXI | 3.19 | 4963.36 | 49.57 | 49.57 | 0.16 | 5027.59 | 0.8 | TAXI | 6.35 | 5724.57 | 57.17 | 57.17 | 0.16 | 5788.62 |
|  | IND | 3.64 | 0.00 | 349.67 | 0.00 | 0.16 | 76.35 |  | IND | 7.11 | 968.58 | 347.14 | 9.67 | 0.16 | 1043.20 |
|  | MFIX | 3.60 | 0.00 | 64.94 | 0.00 | 0.16 | 72.62 |  | MFIX | 7.00 | 1210.95 | 72.18 | 12.09 | 0.16 | 1281.69 |
|  | MHALF | 3.61 | 0.00 | 32.00 | 0.00 | 0.16 | 72.48 |  | MHALF | 7.14 | 1047.74 | 52.76 | 10.46 | 0.16 | 1119.64 |
| 0.6 | TAXI | 2.64 | 10102.10 | 100.89 | 100.89 | 0.17 | 10155.81 | 0.6 | TAXI | 5.13 | 10132.15 | 101.19 | 101.19 | 0.17 | 10184.44 |
|  | IND | 3.58 | 0.00 | 329.18 | 0.00 | 0.18 | 74.89 |  | IND | 7.03 | 775.28 | 327.38 | 7.74 | 0.18 | 848.89 |
|  | MFIX | 3.41 | 0.00 | 113.19 | 0.00 | 0.17 | 69.26 |  | MFIX | 6.86 | 713.18 | 112.56 | 7.12 | 0.17 | 782.92 |
|  | MHALF | 3.57 | 0.00 | 53.07 | 0.00 | 0.18 | 72.00 |  | MHALF | 7.05 | 870.47 | 59.88 | 8.69 | 0.18 | 941.55 |
| 0.4 | TAXI | 1.94 | 14873.09 | 148.54 | 148.54 | 0.19 | 14913.35 | 0.4 | TAXI | 3.70 | 14873.09 | 148.54 | 148.54 | 0.18 | 14911.57 |
|  | IND | 3.42 | 0.00 | 302.63 | 0.00 | 0.19 | 71.40 |  | IND | 6.81 | 499.83 | 300.63 | 4.99 | 0.19 | 570.91 |
|  | MFIX | 3.28 | 0.00 | 162.81 | 0.00 | 0.18 | 67.16 |  | MFIX | 6.53 | 327.55 | 162.14 | 3.27 | 0.19 | 394.43 |
|  | MHALF | 3.36 | 0.00 | 58.97 | 0.00 | 0.19 | 67.80 |  | MHALF | 6.72 | 456.81 | 68.69 | 4.56 | 0.19 | 524.69 |
| 0.2 | TAXI | 1.16 | 19846.46 | 198.21 | 198.21 | 0.23 | 19871.59 | 0.2 | TAXI | 2.22 | 19846.46 | 198.21 | 198.21 | 0.22 | 19870.60 |
|  | IND | 3.21 | 0.00 | 271.83 | 0.00 | 0.22 | 66.97 |  | IND | 6.40 | 182.31 | 271.85 | 1.82 | 0.22 | 248.99 |
|  | MFIX | 3.13 | 0.00 | 211.62 | 0.00 | 0.21 | 64.71 |  | MFIX | 6.32 | 218.37 | 208.59 | 2.18 | 0.21 | 283.67 |
|  | MHALF | 3.12 | 0.00 | 71.77 | 0.00 | 0.21 | 63.11 |  | MHALF | 6.22 | 219.40 | 78.57 | 2.19 | 0.21 | 282.40 |



| Proporti on Fixed | Strategy | Travel | $\begin{aligned} & \text { Taxi } \\ & \text { Cost } \\ & \hline \end{aligned}$ | Dissmila rity | $\begin{aligned} & \text { \# Taxi } \\ & \text { Trips } \\ & \hline \end{aligned}$ | Travel/R equest | $\begin{array}{r} \text { Total } \\ \text { Cost } \\ \hline \end{array}$ | $\begin{array}{\|l} \text { Proporti } \\ \text { on Fixed } \end{array}$ | Strategy | Travel | $\begin{aligned} & \text { Taxi } \\ & \text { Cost } \\ & \hline \end{aligned}$ | Dissmila rity | $\begin{aligned} & \text { \# Taxi } \\ & \text { Trips } \\ & \hline \end{aligned}$ | Travel/R equest | Total Cost |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.8 | TAXI | 3.21 | 4963.36 | 49.57 | 49.57 | 0.16 | 9984.51 | 0.8 | TAXI | 6.41 | 5724.57 | 57.17 | 57.17 | 0.16 | 11505.69 |
|  | IND | 3.63 | 0.00 | 356.59 | 0.00 | 0.16 | 35731.55 |  | IND | 7.14 | 1138.83 | 354.36 | 11.37 | 0.16 | 36646.26 |
|  | MFIX | 3.58 | 0.00 | 49.57 | 0.00 | 0.16 | 5028.68 |  | MFIX | 7.03 | 1384.21 | 57.17 | 13.82 | 0.16 | 7171.46 |
|  | MHALF | 3.71 | 0.00 | 12.72 | 0.00 | 0.17 | 1346.11 |  | MHALF | 7.24 | 1371.22 | 35.98 | 13.69 | 0.17 | 5041.67 |
| 0.6 | TAXI | 2.63 | 10102.10 | 100.89 | 100.89 | 0.17 | 20243.74 | 0.6 | TAXI | 5.25 | 10132.15 | 101.19 | 101.19 | 0.17 | 20303.69 |
|  | IND | 3.53 | 0.00 | 331.84 | 0.00 | 0.18 | 33254.60 |  | IND | 6.98 | 701.17 | 328.71 | 7.00 | 0.18 | 33641.96 |
|  | MFIX | 3.52 | 0.00 | 100.89 | 0.00 | 0.17 | 10159.39 |  | MFIX | 6.90 | 828.33 | 101.19 | 8.27 | 0.18 | 11016.37 |
|  | MHALF | 3.78 | 0.00 | 23.79 | 0.00 | 0.19 | 2454.62 |  | MHALF | 7.19 | 1099.84 | 41.48 | 10.98 | 0.18 | 5319.76 |
| 0.4 | TAXI | 1.87 | 14873.09 | 148.54 | 148.54 | 0.18 | 29764.56 | 0.4 | TAXI | 3.79 | 14873.09 | 148.54 | 148.54 | 0.18 | 29765.04 |
|  | IND | 3.42 | 0.00 | 304.67 | 0.00 | 0.19 | 30535.34 |  | IND | 6.72 | 454.77 | 302.92 | 4.54 | 0.19 | 30814.00 |
|  | MFIX | 3.42 | 0.00 | 148.54 | 0.00 | 0.19 | 14922.31 |  | MFIX | 6.69 | 437.73 | 148.54 | 4.37 | 0.19 | 15358.64 |
|  | MHALF | 3.56 | 0.00 | 36.02 | 0.00 | 0.20 | 3673.26 |  | MHALF | 6.99 | 796.36 | 47.27 | 7.95 | 0.20 | 5593.22 |
| 0.2 | TAXI | 1.14 | 19846.46 | 198.21 | 198.21 | 0.23 | 39690.36 | 0.2 | TAXI | 2.25 | 19846.46 | 198.21 | 198.21 | 0.23 | 39689.96 |
|  | IND | 3.20 | 0.00 | 272.97 | 0.00 | 0.21 | 27360.92 |  | IND | 6.38 | 224.38 | 272.34 | 2.24 | 0.22 | 27522.17 |
|  | MFIX | 3.27 | 0.00 | 198.21 | 0.00 | 0.22 | 19886.45 |  | MFIX | 6.46 | 239.41 | 198.21 | 2.39 | 0.22 | 20125.01 |
|  | MHALF | 3.31 | 0.00 | 48.36 | 0.00 | 0.22 | 4902.20 |  | MHALF | 6.52 | 359.63 | 53.54 | 3.59 | 0.22 | 5778.80 |


| \#Custo | :500; | ehicl | s: 20; $\alpha$ t=1 | 1, $\boldsymbol{0 .}^{\text {¢ }} \mathrm{f}=0.5$ | 0_ ${ }^{\text {l }}$ | as $=$ |  | \#Custo | s: 500; | \#Vehic | s: 10; $\boldsymbol{\alpha} \mathbf{t = 1}$ | , oo_f=0.5 | ao_ ${ }^{\text {l }}$ | , ${ }^{\text {s }}=$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Proporti on Fixed | Strategy | Travel | Taxi <br> Cost | Dissmila rity | $\begin{aligned} & \text { \# Taxi } \\ & \text { Trips } \\ & \hline \end{aligned}$ | Travel/R equest | Total <br> Cost | Proporti on Fixed | Strategy | Travel | Taxi <br> Cost | Dissmila rity | $\begin{aligned} & \text { \# Taxi } \\ & \text { Trips } \\ & \hline \end{aligned}$ | Travel/R equest | Total <br> Cost |
| 0.8 | TAXI | 3.21 | 31.14 | 49.57 | 49.57 | 0.16 | 5052.31 | 0.8 | TAXI | 6.26 | 36.16 | 57.17 | 57.17 | 0.16 | 5815.79 |
|  | IND | 3.45 | 8.62 | 351.70 | 12.53 | 0.16 | 35247.71 |  | IND | 6.73 | 12.99 | 353.80 | 19.03 | 0.16 | 35460.30 |
|  | MFIX | 3.55 | 2.37 | 49.57 | 3.43 | 0.16 | 5030.38 |  | MFIX | 6.87 | 9.45 | 57.17 | 14.19 | 0.16 | 5795.20 |
|  | MHALF | 3.67 | 0.50 | 12.72 | 0.72 | 0.16 | 1345.81 |  | MHALF | 7.28 | 9.17 | 35.98 | 13.88 | 0.17 | 3679.95 |
| 0.6 | TAXI | 2.70 | 63.55 | 100.89 | 100.89 | 0.18 | 10206.48 | 0.6 | TAXI | 5.14 | 63.74 | 101.19 | 101.19 | 0.17 | 10234.18 |
|  | IND | 3.30 | 10.27 | 327.90 | 14.94 | 0.17 | 32866.21 |  | IND | 6.46 | 12.53 | 328.73 | 18.27 | 0.17 | 32950.16 |
|  | MFIX | 3.42 | 4.35 | 100.89 | 6.34 | 0.17 | 10161.67 |  | MFIX | 6.76 | 8.77 | 101.19 | 13.03 | 0.17 | 10195.34 |
|  | MHALF | 3.72 | 0.81 | 23.79 | 1.17 | 0.19 | 2454.25 |  | MHALF | 7.16 | 7.91 | 41.48 | 11.88 | 0.18 | 4227.55 |
| 0.4 | TAXI | 1.90 | 93.37 | 148.54 | 148.54 | 0.18 | 14985.33 | 0.4 | TAXI | 3.73 | 93.37 | 148.54 | 148.54 | 0.18 | 14984.68 |
|  | IND | 3.13 | 10.89 | 302.23 | 15.87 | 0.18 | 30296.40 |  | IND | 6.15 | 11.64 | 301.84 | 16.97 | 0.18 | 30257.15 |
|  | MFIX | 3.11 | 7.31 | 148.54 | 10.68 | 0.18 | 14923.56 |  | MFIX | 6.33 | 8.88 | 148.54 | 13.03 | 0.19 | 14926.16 |
|  | MHALF | 3.49 | 0.89 | 36.02 | 1.30 | 0.20 | 3672.69 |  | MHALF | 6.92 | 5.71 | 47.27 | 8.50 | 0.20 | 4801.94 |
| 0.2 | TAXI | 1.14 | 124.57 | 198.21 | 198.21 | 0.23 | 19968.27 | 0.2 | TAXI | 2.21 | 124.57 | 198.21 | 198.21 | 0.22 | 19967.66 |
|  | IND | 2.86 | 11.64 | 271.72 | 16.95 | 0.20 | 27240.90 |  | IND | 5.65 | 11.87 | 271.81 | 17.30 | 0.20 | 27249.35 |
|  | MFIX | 2.92 | 10.51 | 198.21 | 15.30 | 0.21 | 19889.82 |  | MFIX | 5.82 | 10.35 | 198.21 | 15.09 | 0.21 | 19889.57 |
|  | MHALF | 3.23 | 1.68 | 48.36 | 2.43 | 0.22 | 4902.25 |  | MHALF | 6.39 | 3.19 | 53.54 | 4.67 | 0.22 | 5421.10 |

From the simulation results presented in the previous section, we get the following observations:
I. Strategy TAXI has the smallest travel distance, because of its inability to use the slack times to accommodate the random requests. When handling the same amount of requests on the fleet, strategy MFIX and MHALF have similar travel distance to that of strategy IND, a near-optimal routing solution. This suggests that our Tabu search is effective in bring the routing solutions to near optimum. This is further implied by the results for travel per request, in which cases the solutions from strategy TAXI, IND, MFIX, and MHALF are close.
II. Strategy TAXI has the largest taxi cost and number of taxi trips, again because of its inflexibility of inserting the random requests into the slacks on the fleet of vehicles.
III. Strategy IND has the largest dissimilarity; strategy MHALF has the lowest dissimilarity. If we schedule the routing for each day independently without a master plan, the routes will become very dissimilar from day to day. Even though we get a near optimal solution in terms of routing efficiency as measured in travel distance and taxi cost, the quality of service as measured in route dissimilarity is poor. If we form master routes with the deterministic requests and a number of random requests of high probability of occurrence, we will create daily routes which are similar from day to day, without scarifying much in routing efficiency.
IV. When the unit cost for route dissimilarity increases (from 0.01 to 100), while the other parameters remain the same, the dissimilarity for strategies with master plans decreases and the routing efficiency (travel distance and/or taxi cost) increases. This is because when we give a higher weight on dissimilarity, the routing solution will favor less dissimilarity, and will trade for that with less routing efficiency. The change in the unit cost for route dissimilarity does not have significant impact on the solutions of strategies TAXI and IND. The reason is that for strategy TAXI, the dissimilarity is contributed by the random requests handled by taxi, which remain the same
with any set of parameters; for strategy IND, there is no master plan to use to construct daily routes, but the dissimilarity is measured against the master plan from strategy MFIX, hence the dissimilarity with IND might even increase when the unit cost of dissimilarity increases.
V. When the fixed unit taxi cost decreases (from 100 to 0.5), while all the other parameters remain, there will be more taxi use represented by number of taxi trips. This implies that as taxi is inexpensive, it becomes a more economical solution to use taxi rather than rerouting or picking up packages by the regular fleet of vehicles.
VI. With the same pool of potential customers and the same amount of vehicles, as the proportion of fixed customers increase, the total travel time and the travel time per request increase, because there are more expected customers to service. For the same reason, the taxi cost or number of taxi trips for IND, MFIX or MHAL will remain 0 or increase. The taxi cost or number of taxi trips for TAXI will decrease because there are less random customers for the TAXI strategy to accommodate. The dissimilarity for TAXI, MFIX, and MHALF will decrease because there are more customers included in the master plan. The dissimilarity for IND will even increase because there is no master plan but there are more expected customers to service.
VII. With the same potential customers, when the fleet size increases, the average per vehicle travel time, the taxi cost, and the taxi trips will decrease because there are more vehicles to handle the requests. The dissimilarity for MHALF will decrease because more customers can be included into the master plan. The other trends of the results discussed above remain the same with different size of fleet.
VIII. When the size of the customer pool increases, the problem size increases, resulting larger total cost. The pattern of the above results remains the same for different size of pool of potential customers.

### 5.4 Simulations and Results with Real-Life Data

A study has also been done using real-life data collected from a leading healthcare provider in Southern California. There are two types of requests in the data set. One is regular daily requests, which needs to be visited every day at a specific time. The other is random requests that need to be handled by taxis. We have compared three strategies with this set of data.

1) MD Routes: Include a customer into the master plan if it is deterministic requests or if the pickup and delivery location of a request has a probability of occurring higher than a threshold (e.g., 10\%). Recourse action (drop the nonoccurring requests and insert the occurring requests) is taken for daily plans.
2) Industry Reroute: Take Kaiser's existing routes as master routes. Recourse action (drop the non-occurring requests and insert the occurring requests) for daily plans.
3) Industry Taxi: Take Kaiser’s existing routes for daily routes. Use Taxi for all the random requests.

It should be noted that Industry's Taxi is the current practice of industry, and is taken by the leading health care provider. In the following simulation of a horizon of 30 days, there are 85 deterministic requests and 100 random requests on each day. On a daily basis, 14 vehicles are used to handle all the requests. The simulation results are shown in Table 5.5. As with random data, $\alpha_{t}$ is the unit fleet cost per hour traveled; $\alpha_{o_{-} f}$ is the fixed cost per trip of taxi; $\alpha_{o_{-} v}$ is the varying cost per hour the taxi traveled; $\alpha_{s}$ is the unit cost per count of dissimilarity. Column "Strategy" lists the three strategies we are comparing. Column "Travel" shows the total distance that a vehicle travels per day on average. Column "Taxi Cost" shows the daily taxi cost. Column "Dissimilarity" shows the average dissimilarity, which is the total number of vehicles used in the daily routes that is different than the one in the master routes. If a taxi is used, then the dissimilarity is increased by one. Column "\#Taxi Trips "shows the total number of daily taxi trips introduced on average. Column "Travel/Requests" shows the distance that a vehicle travels to service a request on a daily basis on average. Column "Total Cost" shows the average daily total cost including travel cost, taxi cost, and cost on dissimilarity. It is the summation of each type of costs weighted by the unit cost of that type.

Table 5.5: Simulation Results with Real-Life Data

| $\alpha \mathrm{t}=1, \alpha_{0} \mathrm{f}=100, \alpha_{-}{ }^{\text {a }}=0.5, \alpha \mathrm{~S}=0.01$; |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Strategy | Travel (hours/day) | Taxi Cost (\$/day) | Dissimilarity (counts/day) | $\begin{array}{r} \text { \# Taxi } \\ \text { Trips } \\ \text { (trips/day) } \end{array}$ | Travel/Request (hours/day) | Total Cost (\$/day) |
| MD Routes | 8.78 | 5221.10 | 148.00 | 52.10 | 0.03 | 5345.50 |
| Industry Reroute | 8.36 | 6223.40 | 164.00 | 62.10 | 0.03 | 6342.10 |
| Industry Taxi | 7.24 | 10023.80 | 200.00 | 100.00 | 0.04 | 10127.20 |
| $\alpha \mathrm{t}=1, \alpha_{-} \mathrm{f}=100, \alpha_{-} \mathrm{c}^{2}=0.5, \alpha \mathrm{~s}=100$; |  |  |  |  |  |  |
| Strategy | Travel (hours/day) | Taxi Cost (\$/day) | Dissimilarity (counts/day) | $\begin{array}{r} \text { \# Taxi } \\ \text { Trips } \\ \text { (trips/day) } \end{array}$ | Travel/Request (hours/day) | Total Cost (\$/day) |
| MD Routes | 8.82 | 5291.13 | 143.40 | 52.77 | 0.03 | 19754.60 |
| Industry Reroute | 8.47 | 6333.60 | 157.23 | 63.17 | 0.03 | 22175.51 |
| Industry Taxi | 7.24 | 10023.77 | 200.00 | 100.00 | 0.04 | 30125.17 |
| $\alpha \mathrm{t}=1, \alpha_{0} \mathrm{f}=0.5, \alpha_{0} \mathrm{l}^{\mathrm{v}=0.5, ~} \boldsymbol{\alpha s}=100$; |  |  |  |  |  |  |
| Strategy | Travel (hours/day) | Taxi Cost (\$/day) | Dissimilarity (counts/day) | $\begin{array}{r} \text { \# Taxi } \\ \text { Trips } \\ \text { (trips/day) } \end{array}$ | Travel/Request (hours/day) | Total Cost (\$/day) |
| MD Routes | 8.13 | 43.60 | 134.23 | 54.57 | 0.03 | 13580.76 |
| Industry Reroute | 8.08 | 49.79 | 152.93 | 64.20 | 0.03 | 15456.24 |
| Industry Taxi | 7.24 | 73.77 | 200.00 | 100.00 | 0.04 | 20175.17 |

The results of the analysis with real-life data shows that our heuristic can improve the routing solution by decreasing the taxi cost and dissimilarity cost. With the current resource of vehicles, the current deterministic requests, and sampling on current data set, our heuristic beats the current industry solution by reducing taxi cost by $45 \%-48 \%$ and reducing dissimilarity by $26 \%-33 \%$, with sensitivity analysis on varying cost parameters. If we compare with the daily routes obtained by applying recourse actions on a master plan taken from the current practice industry, our heuristic reduces taxi cost by $16 \%-17 \%$ and it reduces dissimilarity by $9 \%-12 \%$ with sensitivity analysis on varying cost parameters.

## 6. Implementation

This project addresses the area of Courier Delivery Services with urgent demand and stochastic customers in multi-trips with variable trip lengths. A typical application of this project is in the healthcare delivery system, i.e., delivery of medical specimen. Freight transportation is a cost effective way to move the packages among the facilities (i.e., hospitals, clinics, and laboratories). Given the fact that a third party courier is the prevailing solution in the current practice, which results in high transportation cost and low level of customer service, better planning and scheduling tools are needed for this industry. In particular, this research focuses on solving the courier delivery scheduling and dispatching problems.

The scheduling heuristics developed in this research are tested on real-world data collected from a leading health care provider in Southern California region. The performance of the proposed heuristics is compared with the performance of the current industry practice and with improved industry practice. The heuristics developed outperforms the existing approaches. The implementation of our heuristics will require suitable programming software tools such as C++, and access to real-world courier delivery data such as distance and/or travel time between facilities, and historical information on the earliest time packages are allowed to be picked up, service time of processing packages, latest time packages are allowed to be picked up and delivered.

## 7. Conclusions

In this study, we consider a real-life Courier Delivery Problem (CDP), a variant of the Multi-trip Vehicle Routing Problem (MtVRP) with uncertainty in customer occurrence and urgency in customer demands. We present a problem formulation with mixed integer programming for an example application of the transportation of medical specimens. We develop an efficient heuristic based on insertion and Tabu search. Our model represents the probabilistic nature of customer occurrence using scenario-based stochastic programming with recourse. We benefit from the simplicity and flexibility of a master plan with daily recourse actions.

Our model first includes a master plan problem which represents the uncertainty in the customer occurrence by the probabilities customers likely to appear and addresses the urgency in delivery time windows by use of the fleet of vehicles in multiple trips. We then define a recourse action of partial rescheduling of routes by omitting non-occurring customers and rescheduling new customers. The master routes created consider efficiency in routing, to represent slack time for accommodating random requests. The daily plans created take into account the efficiency in routing, efficiency in alternative third party courier, as well as route similarities to boost the quality of service. To solve large size problems of the model, we develop a heuristic based on insertion and Tabu search.

We explore experimentally the sensitivity of our heuristic on randomly generated problems and a real-life problem collected from the industry. Experiments on randomly generated problems include sensitivity analysis in varying problem size, customer uncertainty scenarios, resource availability and cost parameters. We compare the quality of the solution with independent daily scheduling, and to an industry standard solution. In the experiments with real-life data, we compare the quality of solution with current industry solution with and without recourse action. Sensitivity analysis on varying cost parameters shows that our heuristic produces a better solution than the current practice by significantly reducing the cost on taxi use and improving route similarity.

## References

Alonso F., Alvarez M.J., and Beasley J.E., A tabu search algorithm for the periodic vehicle routing problem with multiple vehicle trips and accessibility restrictions, Journal of the Operational Research Society, 59, 963-976, 2008.

Angelelli E. and Speranza M.G., The periodic vehicle routing problem with intermediate facilities, European Journal of Operational Research, 137, 233-247, 2002.

Astion ML, Shojania KG, Hamill TR, Kim S, and Ng V.L., Classifying laboratory incident reports to identify problems that jeopardize patient safety, Am J Clin Pathol, 120, 18-26, 2003.

Azi N., Gendreau M., and Potvin J., An exact algorithm for a single-vehicle routing problem with time windows and multiple routes, European Journal of Operational Research, 178,755-766, 2006.

Bachouch R.B., Guinet A. and Hajri-Gabouj S., A model for scheduling drug deliveries in a French homecare, International Conference on Industrial Engineering and Systems Management (IESM), Montreal, Canada, May 13-15, 2009.

Baker B. M. and M. A. Ayechew, A genetic algorithm for the vehicle routing problem, Computers \& Operations Research, Vol. 30, No. 5, pp. 787 - 800, April, 2003.

Barenfanger J., Drake C., and Kacich G., Clinical and financial benefits of rapid bacterial identification and antimicrobial susceptibility testing, J Clin Microbiol, 37, 1415-8, 1999.

Begur S.V., Miller D.M., and Weaver J.R., An integrated spatial DSS for scheduling and routing Home-Health-Care nurses, Interfaces, 27, 35-48, 1997.

Ben-Tal A. and A. Nemirovski, Robust convex optimization, Mathematics of Operations Research, 23(4):769-805, 1998.

Bertels S. and Fahle T., A hybrid setup for a hybrid scenario: combining heuristics for the home health care problem, Computers and Operations Research, 33, 2866-2890, 2006.

Bertsimas D., Jaillet P., and Odoni A.R., A priori optimization, Operations Research, 38, 1019-1033, 1990.

Bertsimas D., A vehicle routing problem with stochastic demand, Operations Research, 40, 574-585, 1992.

Bertsimas D. and D. Simchi-levi, "A new generation of vehicle routing research: robust algorithms, addressing uncertainty", Operations Research, vol. 44, no. 2, pp. 286-304, Mar. 1996.

Brandao J. and Mercer A., A tabu search algorithm for the multi-trip vehicle routing and scheduling problem, European Journal of Operational Research, 100, 180-191, 1997.

Brandao J. and Mercer A., The multi-trip vehicle routing problem, The Journal of the Operational Research Society, 49, 799-805, 1998.

Breedam A. V., Improvement Heuristics for the Vehicle Routing Problem Based on Simulated Annealing, European Journal of Operational Research, Vol. 86, No. 3, pp. 480-490, Nov. 1995.
R. Carraway, T. Morin, and H. Moskowitz, Generalized dynamic programming for stochastic combinatorial optimization, Operations Research, 37(5):819-829, 1989.

Clarke G. and Wright J.W., Scheduling of vehicles from a central depot to a number of delivery points, Operations Research, 12, 568-581, 1964.

Dror M., Modeling vehicle routing with uncertain demands as a stochastic program: properties of the corresponding solution, European Journal of Operational Research, vol. 64, pp. 432-441, 1993.

Dror M., G. Laporte, and P. Trudeau, Vehicle routing with stochastic demands: properties and solution framework, Transportation Science, vol. 23, no. 3, pp. 166-176, Aug. 1989.

Dror M. and P. Trudeau, Stochastic vehicle routing with modified savings algorithm, European Journal of Operational Research, vol. 23, pp. 228-235, 1986.

Fisher, M., Vehicle routing. In Ball M., Magnanti T., Monma C., and Nemhauser G. (Eds.), Network Routing, Handbooks in Operations Research and Management Science, 8, 1-33, Amsterdam, Elsevier Science, 1995.

Francis P. and Smilowitz K., Modeling techniques for periodic vehicle routing problems, Transportation Research Part B: Methodological, 40, 872-884, 2006.

Ganesh K. and Narendran T.T., CLASH: A heuristic to solve vehicle routing problem with delivery, pick-up and time windows, International Journal of Services and Operations Management, 3, 460-477, 2007.

Gendreau M., H. Alain, and G. Laporte, A Tabu search heuristic for the vehicle routing problem, Management Science, Vol. 40, No. 10, Oct. 1994.

Gendreau M., G. Laporte, and R. Seguin, Stochastic vehicle routing, European Journal of Operational Research, 88 (1): 3-12, 1996.

Gillett B.E. and Miller L.R., A heuristic algorithm for the vehicle dispatch problem, Operations Research, 22, 340-349, 1974.

Groër C., Golden B., and Wasil E., The consistent vehicle routing problem, Article in Press, Manufacturing and Service Operations Management, 2008.

Haughton M., Quantifying the benefits of route reoptimization under stochastic customer demands, Journal of the Operational Research Society, 51, 320-322, 2000.

Haughton M. and Stenger A., Modeling the customer service performance of fixed routes delivery systems under stochastic demands. Journal of Business Logistics, 9, 155-172, 1998.

Hemmelmayr, V.C., Doerner, K.F., Hartl, R.F., A variable neighborhood search heuristic for periodic routing problems, European Journal of Operational Research, 195, 791-802, 2009.

Holland L.L., Smith L.L., and Blick K.E., Reducing laboratory turnaround time outliers can reduce emergency department patient length of stay: an 11-hospital study, Am J ClinPathol124, 672-4, 2005.

Jaillet P., A priori solution of a travelling salesman problem in which a random subset of customers are visited, Operations Research, 36, 929-936, 1988.

Jarrett P., An analysis of international health care logistics: The benefits and implications of implementing just-in-time systems in the health care industry, Leadership in Health Services, 19, 1-10, 2006.

Jezequel.A, Probabilistic vehicle routing problems, Master's thesis, Department of Civil Engineering, Massachusetts Institute of Technology, 1985.

Kao.E., A preference order dynamic program for a stochastic travelling salesman problem, Operations Research, 26:1033-1045, 1978.

Kellerer.H., U. Pferschy, and D. Pisinger, Knapsack Problems, Springer Verlag Berlin Heidelberg, 2004

Lagodimos A.G. and Mihiotis A.N., Overtime vs. regular shift planning decisions in packing shops, International Journal of Production Economics, 101, 246-258, 2006.

Laporte G. and Osman I., Routing problems: a bibliography, Annals of Operations Research, 61, 227-262, 1995.

Laporte, G., F. Louveaux, and H. Mercure, The vehicle routing problem with stochastic travel times, Transportation Science, 26, 161-170, 1992.

Lambert V., G.Laporte, and F. Louveaux, Designing collection routes through bank branches, Computers and Operations Research, 20:783-791, 1993.

Lin S., Computer solutions of the traveling salesman problem. Bell System Technical Journal, 44,2245-2269, 1965.

Meller, R., Pazour, J., Thomas, L., Mason, S., Root, S., and Churchill, W., White paper on hospital pharmacy unit-does acquisition and the case for the third-party repackaging option, Report Series 08-04, Center for Innovation in Healthcare Logistics, Fayetteville, AR, 2008.

Merzifonluoglu Y., Geunes J., and Romeijn H.E., Integrated capacity, demand, and production planning with subcontracting and overtime options, Naval Research Logistics, 54(4), 433-447, 2007.

Modares A., S. Somhom, T. Enkawa, A self-organizing neural network approach for multiple traveling salesman and vehicle routing problems, International Transactions in Operational Research, Vol. 6, Iss. 6, pp. 591-606, Nov 1999.

Nicholson, L., Vakharia, A.J., and Erenguc, S.S., Outsourcing inventory management decisions in healthcare: models and application, European Journal of Operational Research, 154, 271-290, 2004.

Osman I.H., Meta strategy simulated annealing and tabu search algorithms for the vehicle routing problem, Annals of Operations Research, 41, 421-451, 1993.

Papageorgiou L.G., Rotstein, G.E. and Shah, N., Strategic supply chain optimization for the pharmaceutical industries, Industrial \& Engineering Chemistry Research, 40, 275286, 2001.

Petch R. J. and Salhi S., A multi-phase constructive heuristic for the vehicle routing problem with multiple trips, Discrete Applied Mathematics, 133(1-3), 69-92, 2003.

Ren Y., M. M. Dessouky, and F. Ordóñez, The multi-shift vehicle routing problem with overtime, Computers \& Operations Research, 37, 1987-1998, 2010.

Ren Y., Vehicle Routing and Resource Allocation for Health Care under Uncertainty, PhD dissertation, 2011.

Salhi S. and Petch R. J., A GA based heuristic for the vehicle routing problem with multiple trips, Journal of Mathematical Modeling and Algorithms, 6(4), 591—613, 2007.

Secomandi N., A rollout policy for the vehicle routing problem with stochastic demands, Operations Research, vol. 49, no. 5, pp. 796-802, 2001.

Secomandi N. and F. Margot, Reoptimization Approaches for the Vehicle-Routing Problem with Stochastic Demands, Operations Research, vol. 57, No. 1, pp. 214-230, January-February 2009.

Shah N., Pharmaceutical supply chains: Key issues and strategies for optimization, Computers and Chemical Engineering, 28, 929-941, 2004.

Shen Z., F. Ordóñez, and M. M. Dessouky, The stochastic vehicle routing problem for minimum unmet demand, Optimization and Logistics Challenges in the Enterprise, Springer Series on Optimization and its Applications, 2009.

Sniezek J. and Bodin L., Cost Models for Vehicle Routing Problems, Proceedings of the $35^{\text {th }}$ Hawaii International Conference on System Sciences, 1403-1414, 2002.

Steeg J. and Schroeder M., A hybrid approach to solve the periodic home health care problem, Operations Research Proceedings 2007, 297-302, Springer, 2007.

Stewart W. and B. Golden, Stochastic vehicle routing: a comprehensive approach, European Journal of Operational Research, vol. 14, pp. 371-385, 1983.

Steindel S.J., and Howanitz P.J., Physician satisfaction and emergency department laboratory test turn around time, Arch Pathol Lab Med, 125, 863-71, 2001.

Steindel S.J., and Novis D.A., Using outlier events to monitor test turnaround time, Arch Pathol Lab Med, 123, 607-14, 1999.

Sungur, I., F. Ordóñez, and M. M. Dessouky, A robust optimization approach for the capacitated vehicle routing problem with demand uncertainty, IIE Transactions 40, 509523, 2008.

Sungur I., F. Ordóñez, M. M. Dessouky, Y. Ren, and H. Zhong, A Model and Algorithm for the Courier Delivery Problem with Uncertainty, Transportation Science, 44, 193-205, 2010.

Taillard E., Laporte G., and Gendreau M., Vehicle routing with multiple use of vehicles, Journal of the Operational Research Society, 47, 1065-1070, 1996.

Thompson P.M. and Psaraftis H.N., Cyclic transfer algorithms for multi-vehicle routing and scheduling problems, Operations Research, 41, 935-946, 1993.

Toth P. and D. Vigo, The vehicle routing problem, SIAM Monographs on Discrete Mathematics and Applications, SIAM Publishing, 2002.

Vissers J.M.H. and Beech, R., Health operations management: patient flow logistics in health care, Oxon: Routledge, 2005.

Waters C.D.J., Vehicle-scheduling problems with uncertainty and omitted customers, The Journal of the Operational Society, 40, 1099-1108, 1989.

Zapfel G., and Bogla M., Multi-period vehicle routing and crew scheduling with outsourcing options, International Journal of Production Economics, Article in Press, 2008.

Zhong, H., R. W. Hall, and M. M. Dessouky, Territory Planning and Vehicle Dispatching, Transportation Science, 41, 74-89, 2007

## Appendix A. Simulation Results on Randomly Generated Data

Figure A.1: Travel Time with 50 Customers

| \# Vehicle =4 | \# Vehicle =2 |
| :---: | :---: |
| Travel Time at $=1, a_{-} \mathrm{f}=100, \mathrm{ao}_{\mathbf{\prime}} \mathbf{v}=\mathbf{0 . 5}$, as=0.01 | Travel Time $a t=1, a_{-} \mathrm{f}=100, a_{0} \quad \mathrm{v}=\mathbf{0 . 5}$, as=0.01 |
| Travel Time $a t=1, a_{-} f=100, a_{-} \quad v=0.5, a s=100$ | Travel Time $a t=1, a a_{-} f=100, a 0 \_v=0.5, a s=100$ |
| Travel Time $a t=1, a_{-} \_\mathbf{f}=0.5, a_{-} \quad v=0.5, a s=100$ | Travel Time $a t=1, a_{0} \_\mathbf{f}=0.5, a_{-} \_v=0.5, a s=100$ |

Figure A.2: Taxi Cost with 50 Customers

| \# Vehicle = 4 | \# Vehicle =2 |
| :---: | :---: |
| Taxi Cost $\mathrm{at}=1, \mathrm{ao}_{-} \mathrm{f}=100, \text { ao_v}=0.5, \mathrm{as}=0.01$  | Taxi Cost $a t=1, a o \_f=100, a 0 \_v=0.5, a s=0.01$  |
| Taxi Cost $a t=1, a_{-} \_=100, a_{-} \_v=0.5, a s=100$  | Taxi Cost $a t=1, a_{-} \_f=100, \text { ao_v }=0.5, a s=100$  |
| Taxi Cost $a t=1, a o \_f=0.5, \text { ao_v }=0.5, a s=100$  | Taxi Cost $\mathrm{at}=1, \mathrm{ao} \_\mathrm{f}=0.5, \mathrm{ao} \_\mathrm{v}=0.5, \mathrm{as}=100$  |

Figure A.3: Dissimilarity with 50 Customers

| \# Vehicle =4 | \# Vehicle =2 |
| :---: | :---: |
|  |  |
|  |  |
| $\begin{gathered} \text { Dissimilarity } \\ \text { at }=1, \text { ao_f }=0.5, \text { ao_ } v=0.5, a s=100 \end{gathered}$  | Dissimilarity $\text { at }=1, \text { ao_ }^{\prime}=0.5, a_{0} \_v=0.5, \text { as }=100$  |

Figure A.4: Number of Taxi Trips with 50 Customers

| \# Vehicle =4 | \# Vehicle =2 |
| :---: | :---: |
| Number of Taxi Trips $\text { at }=1, a_{0} \_f=100, a 0 \_v=0.5, \text { as }=0.01$  | Number of Taxi Trips $\text { at }=1, a_{0} \_f=100, \text { ao_v }=0.5, \text { as }=0.01$  |
| Number of Taxi Trips $a t=1, a_{-} \mathrm{f}=100, a_{-} \mathrm{v}=0.5$, as $=100$ | Number of Taxi Trips $\text { at }=1, a_{0} \_f=100, a_{-} \_v=0.5, \text { as }=100$  |
| Number of Taxi Trips $\mathrm{at}=1, \text { ao_f }=0.5, \text { ao_v }=0.5, \mathrm{as}=100$  | Number of Taxi Trips $\mathrm{at}=1, \text { ao_f }=0.5, \text { ao_v }=0.5, \mathrm{as}=100$  |

Figure A.5: Travel Time per Request with 50 Customers

| \# Vehicle =4 | \# Vehicle =2 |
| :---: | :---: |
| Travel Time per Request $a t=1, a 0 \_f=100$, ao_v $=0.5$, as=0.01 | Travel Time per Request $a t=1, a_{-} \mathrm{f}=100, a_{0} \quad \mathrm{v}=0.5$, $a s=0.01$ |
| Travel Time per Request $a t=1, a_{-} \_=100, a_{-} \quad v=0.5, a s=100$ | Travel Time per Request at $=1$, ao_f $=100, a_{0} \quad v=0.5$, as $=100$ |
| Travel Time per Request $a t=1, a_{0} \_f=0.5, a_{0} \quad v=0.5$, $a s=100$ | Travel Time per Request $a t=1, a_{0} \_f=0.5, a_{0} \quad v=0.5$, as=100 |

Figure A.6: Total Daily Cost with 50 Customers

| \# Vehicle =4 | \# Vehicle =2 |
| :---: | :---: |
| Total Daily Cost $a t=1, a_{-} f=100, a_{0} \quad v=0.5, a s=0.01$ | Total Daily Cost $a t=1, a_{-} \quad f=100, a_{0} \quad v=0.5, a s=0.01$ |
| Total Daily Cost $a t=1, a_{-} \_=100, a o_{-} v=0.5, a s=100$ | Total Daily Cost $a t=1, a_{-} \mathrm{f}=100, \mathrm{ao}_{-} \mathrm{v}=\mathbf{0 . 5}$, as=100 |
| Total Daily Cost $\mathrm{at}=1, \text { ao_ } \mathrm{f}=0.5, \text { ao_v }=0.5, \mathrm{as}=100$  | Total Daily Cost $\mathrm{at}=1, \text { ao_f }=0.5, \text { ao_ } \mathrm{v}=0.5, \mathrm{as}=100$  |

Figure A.7: Travel Time with 100 Customers

| \# Vehicle =8 | \# Vehicle = 4 |
| :---: | :---: |
| Travel Time $\text { at }=1, a_{0} \_f=100, a 0 \_v=0.5, a s=0.01$  | Travel Time $a t=1, a o \_f=100, a 0 \_v=0.5, a s=0.01$  |
| Travel Time $a t=1, a_{-} \quad f=100, a a_{-} v=0.5, a s=100$ | Travel Time $a t=1, a_{0} \_f=100, a a_{-} v=0.5, a s=100$  |
| Travel Time $a t=1, a_{0} \_f=0.5, a_{0} \_v=0.5, a s=100$  | Travel Time $a t=1, a o \_f=0.5, \text { ao_v }=0.5, a s=100$  |

Figure A.8: Taxi Cost with 100 Customers

| \# Vehicle =8 | \# Vehicle = 4 |
| :---: | :---: |
| Taxi Cost $a t=1, a o_{-} f=100, a_{0} \quad v=0.5, a s=0.01$ | Taxi Cost $a t=1, a a_{-} f=100, a_{-} \quad v=0.5, a s=0.01$ |
| Taxi Cost at $=1, a_{0} \_\mathbf{f}=100, a_{-} \quad \mathbf{v}=0.5$, as $=100$ | Taxi Cost $a t=1, a_{-} \_=100, a o_{-} v=0.5, a s=100$ |
| Taxi Cost $a t=1, a_{0} \_f=0.5, a o \_v=0.5, a s=100$  | > Taxi Cost at $=1$, ao_f $=0.5$, ao_ $^{2}=0.5, ~ a s=100$  |

Figure A.9: Dissimilarity with 100 Customers

| \# Vehicle =8 | \# Vehicle = 4 |
| :---: | :---: |
|  |  |
| Dissimilarity $a t=1, a_{-} f=100, a_{-} \quad v=0.5, a s=100$ | Dissimilarity $a t=1, a_{-} \mathrm{f}=100, \mathrm{ao}_{-} \mathrm{v}=0.5$, $\mathrm{as}=100$ |
| Dissimilarity $a t=1, \text { ao_f }=0.5, \text { ao_v }=0.5, a s=100$  | Dissimilarity $\text { at }=1, \text { ao_f }=0.5, \text { ao_v }=0.5, \text { as }=100$  |

Figure A.10: Number of Taxi Trips with 100 Customers

| \# Vehicle =8 | \# Vehicle = 4 |
| :---: | :---: |
| Number of Taxi Trips $\text { at }=1, a_{0} \_f=100, \text { ao_v }=0.5, \text { as }=0.01$  | Number of Taxi Trips $\text { at }=1, \text { ao_f }=100, \text { ao_v }=0.5 \text {, as }=0.01$  |
| Number of Taxi Trips $a t=1, a_{-} \_=100, a a_{-} v=0.5, a s=100$ | Number of Taxi Trips at=1, ao_f=100, ao_v=0.5, as=100  |
| Number of Taxi Trips $a t=1, a 0 \_f=0.5, a o \_v=0.5, a s=100$  | Number of Taxi Trips $a t=1, a 0 \_f=0.5, \text { ao_v }=0.5, a s=100$  |

Figure A.11: Travel Time per Request with 100 Customers

| \# Vehicle =8 | \# Vehicle =4 |
| :---: | :---: |
| Travel Time per Request $a t=1, a 0 \_f=100, a_{0} \quad v=0.5$, as=0.01 | Travel Time per Request $a t=1, a_{-} \mathrm{f}=100, a_{0} \quad \mathrm{v}=0.5$, as=0.01 |
| Travel Time per Request $a t=1, a_{-} \_=100, a_{-} \quad v=0.5, a s=100$ | Travel Time per Request $a t=1, a a_{-} f=100, a_{-} \quad v=0.5, a s=100$ |
| Travel Time per Request $a t=1, a_{0} \_f=0.5, a_{0} \_v=0.5$, as $=100$ | Travel Time per Request $a t=1, a_{0} \_\mathbf{f}=0.5, a_{0} \quad v=0.5, a s=100$ |

Figure A.12: Total Daily Cost with 100 Customers

| \# Vehicle =8 | \# Vehicle =4 |
| :---: | :---: |
| Total Daily Cost $a t=1, a 0 \_f=100, a 0 \_v=0.5$, as=0.01 | Total Daily Cost $a t=1, a 0 \_f=100, a 0 \_v=0.5$, as=0.01 |
| Total Daily Cost $a t=1, a o \_f=100, a a_{-} v=0.5$, as=100 | Total Daily Cost $a t=1, a a_{-} f=100, a o_{-} v=0.5, a s=100$ |
| Total Daily Cost $a t=1, a_{-} \_=0.5, a_{-} \quad v=0.5, a s=100$ | Total Daily Cost $a t=1, a_{0} \_\mathbf{f}=0.5, a_{n} \quad v=0.5$, $a s=100$ |

Figure A.13: Travel Time with 500 Customers

| \# Vehicle $=20$ | \# Vehicle $=10$ |
| :---: | :---: |
| Travel Time $a t=1, a_{0} f=100, a_{0} \quad v=0.5, a s=0.01$ | Travel Time $a t=1, a o \_f=100, a o \_v=0.5, a s=0.01$  |
| Travel Time $a t=1, a o \_f=100, a a_{-} v=0.5, a s=100$ | Travel Time $a t=1, a a_{-} f=100$, ao_v $=0.5$, $a s=100$ |
| Travel Time $\mathrm{at}=1, \mathrm{ao}_{\mathrm{o}} \mathrm{f}=0.5, \mathrm{ao}_{-} \mathrm{v}=0.5, \mathrm{as}=100$  | Travel Time $\mathrm{at}=1, \mathrm{ao} \_\mathrm{f}=0.5, \mathrm{ao} \_\mathrm{v}=0.5, \mathrm{as}=100$  |

Figure A.14: Taxi Cost with 500 Customers

| \# Vehicle $=20$ | \# Vehicle $=10$ |
| :---: | :---: |
| Taxi Cost $\mathrm{at}=1, \text { ao_f }=100, \text { ao_v }=0.5, a \mathrm{a}=0.01$  | Taxi Cost $\mathrm{at}=1, \text { ao_f }=100, \text { ao_v }=0.5, a \mathrm{a}=0.01$  |
| Taxi Cost $a t=1, a_{-} \_f=100, a o_{-} v=0.5, a s=100$  | Taxi Cost $a t=1, a o f=100, a o \_v=0.5, a s=100$  |
| Taxi Cost $a t=1, a_{0} \_f=0.5, a_{n} \quad v=0.5, a s=100$ | Taxi Cost $a t=1, a 0 \_f=0.5, a o \_v=0.5, a s=100$  |

Figure A.15: Dissimilarity with 500 Customers

| \# Vehicle $=20$ | \# Vehicle =10 |
| :---: | :---: |
| Dissimilarity $a t=1, a o \_f=100, a 0 \_v=0.5$, $a s=0.01$ |  |
| Dissimilarity <br> $a t=1, a o \_f=100, a a_{-} v=0.5, a s=100$ | Dissimilarity <br> $a t=1, a o f=100, a o_{-} v=0.5, a s=100$ |
| Dissimilarity $\mathrm{at}=1, \mathrm{ao} \_\mathrm{f}=0.5, \mathrm{ao} \_\mathrm{v}=0.5, \mathrm{as}=100$ | Dissimilarity $\mathrm{at}=1, \mathrm{ao}_{-} \mathrm{f}=0.5, \mathrm{ao} \_\mathrm{v}=0.5, \mathrm{as}=100$  |

Figure A.16: Number of Taxi Trips with 500 Customers

| \# Vehicle $=20$ | \# Vehicle $=10$ |
| :---: | :---: |
| Number of Taxi Trips $\text { at }=1, \text { ao_f }=100, \text { ao_v }=0.5, \text { as }=0.01$  | Number of Taxi Trips $\text { at }=1, a_{0} \_f=100, a_{0} \_v=0.5, \text { as }=0.01$  |
| Number of Taxi Trips $a t=1, a_{-} \_=100, a_{-} \quad v=0.5, a s=100$ | Number of Taxi Trips $a t=1, a_{-} f=100, a_{-} \quad v=0.5$, $a s=100$ |
| Number of Taxi Trips $\mathrm{at}=1, \text { ao_f=0.5, ao_v=0.5, as=100 }$  | Number of Taxi Trips $a t=1, a_{-} \_f=0.5, a_{0} \_v=0.5, a s=100$  |

Figure A.17: Travel Time per Request with 500 Customers

| \# Vehicle =20 | \# Vehicle =10 |
| :---: | :---: |
| Travel Time per Request $a t=1, a a_{-} f=100, a_{0} \quad v=0.5$, $a s=0.01$ | Travel Time per Request $a t=1, a a_{-} f=100, a_{0} \quad v=0.5, a s=0.01$ |
| Travel Time per Request $a t=1, a o \_f=100, a o_{-} v=0.5, a s=100$ | Travel Time per Request $a t=1, a_{-} \_f=100, a_{-} \quad v=0.5, a s=100$ |
| Travel Time per Request $a t=1, a_{-} \_=0.5, a_{-} \quad v=0.5, a s=100$ | Travel Time per Request $a t=1, a_{0} \_f=0.5, a_{-} \_v=0.5$, as $=100$ |

Figure A.18: Total Daily Cost with 500 Customers

| \# Vehicle =20 | \# Vehicle =10 |
| :---: | :---: |
| Total Daily Cost $\text { at }=1, a_{0} \_f=100, \text { ao_v }=0.5, \text { as }=0.01$  | Total Daily Cost at $=1, a_{-} \quad \mathrm{f}=100, \mathrm{ao}_{-} \mathrm{v}=0.5$, as=0.01 |
| Total Daily Cost $a t=1, a o_{-} f=100, a o_{-} v=0.5$, $a s=100$ | Total Daily Cost $a t=1, a_{-} \_=100, a o_{-} v=0.5, a s=100$ |
| Total Daily Cost $\mathrm{at}=1, \mathrm{ao} \_\mathrm{f}=0.5, \mathrm{ao} \_\mathrm{v}=0.5$, $\mathrm{as}=100$ | Total Daily Cost $a t=1, a 0 \_f=0.5, a 0 \_v=0.5, a s=100$ |

