Development of Methods for Handling Empty Containers with Applications in the Los Angeles/Long Beach Port Area

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Abstract

The Los Angeles/Long Beach (LA/LB) port complex is the intermodal gateway to Pacific Rim trade and the busiest container port complex in the United States. Comprising of fourteen individually gated terminals, during 1999 alone, the combined ports handled 8.2 million 20-foot equivalent units (TEUs) containers. This figure implies that almost 4.43 million full containers were handled during 1999 in the LA/LB port complex (at the rate of 1.85 TEU/container).

Usually the arriving loaded containers at ports are picked up and transported by trucks to their destinations. Having been unloaded at the importers, the emptied containers are returned to the port. At the same time, empties are picked up by trucks from the ports and brought to the export firms, where they are loaded with export goods. The loaded containers are then, transported to the port to be loaded on the ship for export. In this procedure the empty containers are handled twice at marine terminals i.e. the first time when they are recycled from importers, and the second time when they are trucked to exporters. It is clear that a system, which facilitates the interchange of empties outside the ports, is not only desirable but also necessary. This system will reduce the truck trips to and from container terminals, and as a consequence, will reduce the traffic congestion around the ports. In addition to saving time for both truckers and port operators, the system will significantly reduce noise and emissions around container terminals.

In this report, the empty container interchange problem is investigated in both deterministic and stochastic transportation environments. In stochastic networks the problem is modeled analytically and optimization techniques are developed. In deterministic environments, the empty container substitution problem, in which the request of one type of containers could be fulfilled with another type, is investigated. The simulation experiments are used to demonstrate the efficiency of the developed optimization techniques and approach.

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Disclosure

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1 Introduction

The Los Angeles and Long Beach (LA/LB) port complex, located in San Pedro Bay, is the largest U.S. ocean freight hub and the busiest container port complex. It consists of fourteen individually gated terminals, and serves as a crucial node in the regional supply chain [2]. Over the period of 1990 to 2000, the growth in container traffic in the LA/LB port was significant. With an average annual growth of 9.2%, this figure surpasses the forecasted annual growth of 6.2%, which had been the basis of the 2020 Seaport Plan and much of the regional economic and infrastructure planning [1, 2]. Assuming a modest 6.2% annual growth, the estimated container traffic in 2020 will be around 28 million TEUs or almost 15.1 million containers. Thus, by 2020 the volume of containers moving through the combined LA/LB ports will be at least three times the current volume [2].

As a consequence of this unanticipated growth, port generated traffic has emerged as a major contributor to regional congestion. Traffic congestion and long queues at the gates of the LA/LB terminals are the main source of air pollution (especially diesel toxins), wasted energy, driver inefficiency, and increasing maintenance cost imposed by the volume of trucks on the roadway [3]. Moreover, a study by the California Highway Patrol in southern California freeways reveals that the main artery to the LA/LB terminals, the I-710 freeway, topped the list of freeway collisions on two measures, the highest proportion of truck-involved collisions, 31%, and truck-caused collisions at 16% [4].

There are numerous ways to improve traffic congestion at ports, and therefore reduce transport times associated with container movements. Options include developing new facilities and expanding current ones, deploying advanced technologies, and improving operational characteristics at ports. The scarcity of land at major ports, however, has made the option of developing new facilities, if not infeasible, significantly costly. Feasible options are therefore those that rely on more intelligent decision making to make current operations more efficient.

The purpose of this research is to investigate a more intelligent way of handling empty containers which will lead to more efficient operations of container transport. In particular we investigate the possibility of interchanging empty containers outside the container terminals in an effort to reduce traffic congestion in and around the container terminals.

At marine terminals, containers are typically handled twice. Once as loaded containers, and the second time recycled as empties. Thus, the empty container movements can be divided into two major categories: import and export movements. The import container movements, demonstrated in Figure 1, can be briefly described as follows: a truck is dispatched to pick-up a loaded import container from the terminal (move 1); the truck then delivers the loaded container to its designated local consignee (move 2); the truck may return without any container if the empty container is not available immediately for pick up (move 3); another truck may be dispatched to pick up the empty container (move 4); the truck returns the emptied container to the terminal (move 5), and goes back to its trucking company (move 6). Note that, moves 3 and 4 in Figure 1 can be eliminated if the truck delivering the loaded container in move 2 returns an emptied container to the terminal.

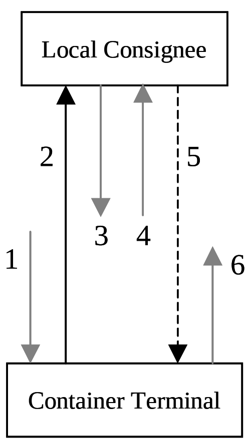


Figure 1: Import container movement.

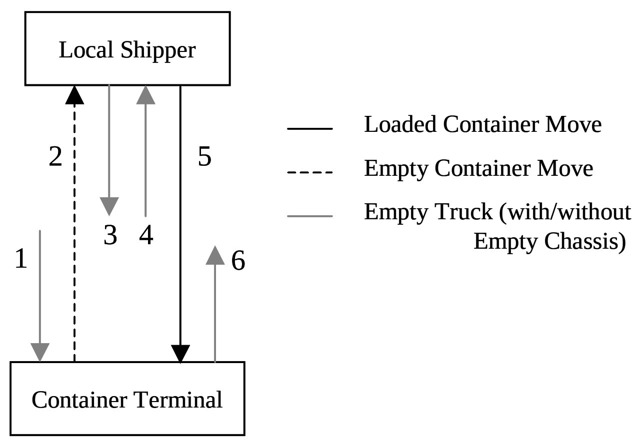


Figure 2: Export container movement.

Likewise, export container movements, shown in Figure 2, are as follows: a truck is dispatched to pick up an empty container (move 1) from the terminal; the empty container is trucked to designated local shipper for loading (move 2); the truck may return without the loaded container if it is not available immediately (move 3); another truck may be dispatched to pick up the loaded container (move 4); the loaded container is returned to the terminal (move 5), and finally the truck goes back to its trucking company (move 6). Similar to import container movements, moves 3 and 4 in Figure 2 can be eliminated if the truck delivering the empty container in move 2 returns the loaded container to the terminal.

It is predicted that by 2020 the number of empties trucked back from local consignees to the LA/LB container terminals would be more than 4,585,000 containers (i.e., 63% of all empties trucked to the terminals). At the same time local shippers will be in need for 1,900,000 empty containers (i.e., 72% of all empties moved by trucks out of the terminals) for loading export goods [1]. These numbers indicate that the local businesses (local consignees and shippers) will be the largest contributors to empty flows in the region, and consequently, to the congestion at and around the LA/LB container terminals.

It is clear that a system, which facilitates the interchange of empties outside the ports, is not only desirable but also necessary. Such a system will be a dual use system for both military and civilian users. The idea of empty container “reuse” consists of using empty import containers for export loads without first returning them to the marine terminal. Generally speaking, two major methodologies can be considered for reusing empty containers (a) depot-direct, and (b) street-turn [1, 5].

In the depot-direct case, in addition to the marine terminals, empty containers are stored, maintained, and interchanged at off-dock container depots. In this methodology, the off-dock empty depots let drivers drop-off or pick-up empty containers without waiting in marine terminal queues. Such depots are usually located very close to the ports. The concept of off-dock empty depot may be more attractive and promising in the long term rather than in the short term. In the short term the concept may be costly to all parties involved. In the long-term, however, congested marine terminals and the high capital cost of expanding on-dock container would justify the higher operating cost of empty depots [1]. In places such as Hong Kong, the shortage of space has already forced operators to shift as many operations as possible off-dock. Most likely similar practices will emerge in other countries, once the major economic and institutional barriers are addressed.

In the street-turn case, the empty container is directly moved from local consignees to local shippers. This movement is represented in Figure 3, where two empty container moves (from the consignee to the terminal and from the terminal to the shipper have been eliminated and have been replaced by a single local move from consignee to shipper, i.e. move 3 in Figure 3).

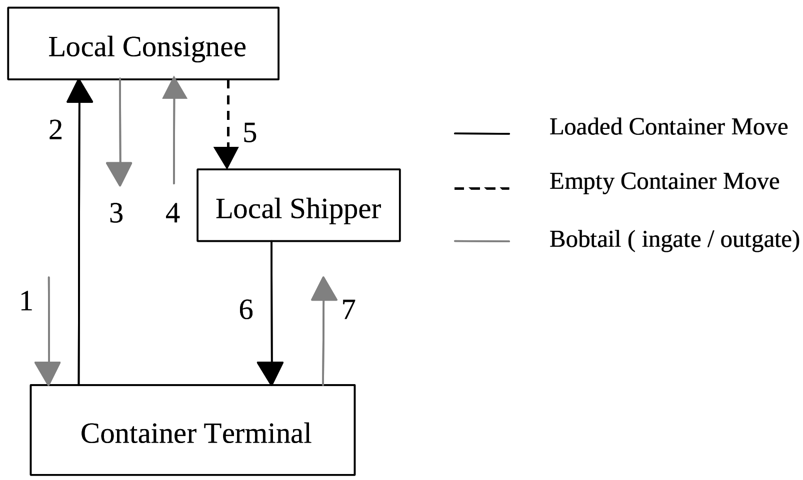


Figure 3: Street-turn empty container reuse.

Despite the importance of the empty container reuse problem, the research efforts in this area have been scant. As noted by Dejax and Crinic, even the work on developing models related to the container transportation problems is very limited [6]. Crainic et al. [7] proposed dynamic and stochastic models for empty container allocation in a land distribution and transportation system. Cheung and Chen [8] formulated the dynamic container allocation problem as a two-stage stochastic network model. The model assists liner operators to allocate their empty containers and consequently reducing their leasing cost and the inventory level at ports. In another related work, Choong et al. [9] addressed the effect of the length of the planning horizon on empty container management. They used the intermodal container-on-barge operation in Mississippi river as the case study to investigate the advantages of using a long rolling horizon.

The empty container allocation problem was also considered and studied by Jula et al [5], [19]. In [5], the authors proposed a model and an optimization technique for the dynamic allocation of empty containers in the LA/LB port and its vicinity, and showed that the empty container reuse can yield a significant reduction in the number and cost of truck trips.

In this research, and in an effort to further reduce the cost of empty container interchange, we investigate the possibility of fulfilling the request of one type of containers with another type.

This problem is sometimes referred to as the multi-commodity empty substitution problem. In addition, in this work the empty container reuse problem in stochastic environments is investigated. The objectives of this research are three-fold:

a) to develop analytical models and optimization techniques that will minimize the cost of empty container interchange in stochastic environments,

b) to develop analytical models and optimization techniques for the multi-commodity empty container substitution problem, and

c) to develop realistic simulation scenarios using past, current and projected data in the Los Angeles/Long Beach port area to evaluate the developed optimization methods in parts “b” and “c” above.

This report is organized as follows. In Section 2, the multi-commodity empty interchange problem with substitution is investigated. The problem is formulated analytically and an optimization technique is developed. In Section 3, the stochastic empty container interchange problem is studied. The problem is modeled as a stochastic problem with recourse. Different optimization techniques are developed and compared. Sections 4 and 5 consider the recommendations for future work and implementation considerations.

2 Multi-commodity Empty Interchange: Modeling and Optimization

In an attempt to further reduce the cost of the empty container allocation problem, this study investigates the substitution between empty containers. The substitution allows the possibility of fulfilling a request for one type of empty containers with another available type.

In this section, we propose models and an optimization algorithm for the empty container allocation problem with substitution in deterministic networks by following a similar procedure as in our earlier work [5], [19]. The developed optimization algorithm divides the problem into dependent and independent parts and applies a branch-and-bound type procedure to the dependent part.

2.1 Container Types and Substitution Rules

This subsection describes the substitution mechanism in the multi-commodity transportation problem. We first define the types of containers followed by the substitution rules between different container types. Throughout this section, we may use the terms “commodity” and “container” interchangeably.

2.1.1 Container types

Containers can be classified into separable types (classes) according to their intended use, external dimension, ownership, etc. For instance, if the types of containers can be determined by only three attributes: purpose, dimensions, and ownership, a type t container can be expressed as

$$\text{type } t = \left\{ \text{ purpose, dimension, ownership } \right\}.$$

The purpose indicates the intended use of containers such as general (dry cargo) or specific purpose (refrigerated, specialized, etc.) containers. Most containers are sized according to the International Standards Organization (ISO). Based on ISO, containers are described in terms of TEU (Twenty-foot Equivalent Units) in order to facilitate comparison of one container system with another. A TEU is 8 feet wide, 8 feet high and 20 feet long container. An FEU is an 8-foot high, 40-foot long container and is equivalent to two TEUs. Containers with height of 9.5 feet are usually referred to as high cube containers. The most widely used containers are general purpose FEU containers. In this report, we will only consider standard dry cargo containers with the following standard dimensions:

$$D_1: 40' \times 8' \times 8.5', \quad D_2: 20' \times 8' \times 8.5', \quad D_3: 40' \times 8' \times 9.5' \tag{1}$$

2.2 Substitution Rules

The substitution rules are the rules specified for substituting each ordered pair of container types. These rules may be symmetric or non-symmetric. For instance, suppose that there are three types of containers,

$$t_1 = \{\text{ General, D}_1\text{, Hanjin }\},\ t_2 = \{\text{ General, D}_2\text{, Hyundai }\},\ \text{and}\ t_3 = \{\text{ General, D}_3\text{, Maersk }\} \tag{2}$$

where D_1, D_2, and D_3 are standard dimensions defined in Subsection 2.1.1. The possible non-symmetric container substitutions between t_1, t_2, and t_3 could be as follows.

- **Non-symmetric in type for (t_1,t_3)**: One request for t_3 could be fulfilled by one supply of t_1, but the reverse substitution is not permitted. This case may happen when certain customers do not accept the high cube containers t_3 due to their facility limitation.
- **Non-symmetric in number for (t_2,t_1)**: One request for t_1 can be fulfilled by two supplies of t_2. However, sometimes, two requests for t_2 can only be satisfied by two supplies of t_1. This case happens when export cargos in a shipper location have two different destinations or when it is desirable to have two small containers because of the weight limitation.

Generally speaking, the substitution rules come from differences in the handling capacity of the loading/unloading facilities, the destination of cargos, the weight of cargos, or the nature of cargos. Other factors may include operational regulations or limitations set forth by freight liners or trucking companies.

Let T be the set of container types and v_k^t be the number of requested empties of type $t \in T$ at shipper k. We assume that the request consists of two parts: (a) exact, and (b) substitutable requests. The exact request for container type t at shipper k, denoted by $v_{k,e}^t$, must be fulfilled by the exact type t, whereas the substitutable request, denoted by $v_{k,s}^t$, can be satisfied by any type. Hence, we have

$$v_k^t = v_{k,e}^t + v_{k,s}^t .$$

We define the extra request at shipper k with respect to an FEU as

$$e_k = \sum_{t \in T} a^t v_{k,s}^t$$

where $1/a^t$ indicates the number of containers of type t needed to substitute one FEU container. For instance, if the extra request is for one container of type t_1, where t_1 is defined in (2), the request can be either substituted by two t_2 containers (i.e., $a^t = 0.5$)or by one t_1 or t_3 (i.e., $a^t = 1$).

Expanding on the above concept, let $r_k^{t_i t_j}$ be the substitution rule coefficient between container type t_i and t_j at demand node k. Then, $1/r_k^{t_i t_j}$ indicates the number of containers of type t_i needed to satisfy one extra request originated from the requests for t_j at k. For instance, Table 1 represents the possible substitution rules between two container types t_1 and t_2 defined in (2) at demand node k.

Table 1: Possible substitution rules between type t_1 and t_2

substitution flows	$r_k^{t_1 t_1}$	$r_k^{t_2 t_1}$	$r_k^{t_1 t_2}$	$r_k^{t_2 t_2}$	substitution rule
$t_1 \to t_2$ & $t_2 \to t_1$	1	0.5	0.5	0.5	symmetric in type
$t_1 \to t_2$ & $t_2 \not\to t_1$	1	0	1	0.5	non-symmetric in type
$t_1 \to 2t_2$ & $2t_2 \to t_1$	1	0.5	1	0.5	symmetric in number

$t_1 \to 2t_2$ & $t_2 \to t_1$	1	1	1	0.5	non-symmetric in number

2.3 Empty Container Allocation Problem

This subsection describes the empty container allocation problem as a typical transportation problem. First the analytical model of the empty container allocation problem without substitution is briefly reviewed. Then, a model and an approximation method are proposed for two-commodity substitution problem. Finally, we develop a model and a decomposition method for substituting multiple types of containers.

2.3.1 Single commodity problem (Problem P1)

When there is no substitution allowed, the multi-commodity allocation problem can be decomposed into a series of single commodity transportation problems for each empty container type. The single commodity transportation problem is presented below, which, hereafter, is referred to as problem **P1**.

Problem **P1:**

$$\text{Minimize:} \qquad \sum_{t\in T}\sum_{i=1}^{m+p}\sum_{j=1}^{n+p} c_{ij}^t x_{ij}^t \tag{3}$$

$$\text{Subject to} \qquad \sum_{j=1}^{n+p} x_{ij}^t = u_i^t, \qquad \forall\ i \in I \cup P,\ t \in T \tag{4}$$

$$\sum_{i=1}^{m+p} x_{ij}^t = v_j^t, \qquad \forall\ j \in J,\ t \in T \tag{5}$$

$$x_{ij}^t \geq 0,\ \text{integer}, \qquad \forall\ i \in I \cup P,\ j \in J,\ t \in T \tag{6}$$

where

I : the set of consignees, $|I| = m$.

J : the set of shippers, $|J| = n$.

P : the set of depots including terminals, $|P| = p$.

T : the set of container types, $|T| = q$.

c_{ij}^t : the cost of transporting a type $t \in T$ container from supply node $i \in I \bigcup P$ to demand node $j \in J \bigcup P$.

x_{ij}^t : the decision variable that represents the number of type $t \in T$ containers transported from supply node $i \in I \bigcup P$ to demand node $j \in J \bigcup P$.

u_i^t : the number of available empties of type $t \in T$ in supply node $i \in I \bigcup P$.

v_j^t : the number of requested empties for type $t \in T$ in demand node $j \in J$.

In problem P1, constraints (4) ensure that the total number of empties moved from each consignee is equal to the number of supply of empties at that location. Constraints (5) guarantee that the number of empties arrived at each shipper is the same as the demand of empties at that location. Finally, constraints (6) are the integer constraints.

In this report, the total number of available containers of type t in supply nodes $I \bigcup P$ is assumed to be greater than or equal to the total number of requested empties of type t in demand nodes J . In other words, we assume that all the demands can be satisfied by internal supplies, rather than exogenous resources. Hence, we have

$$\sum_{i=1}^{m+p} u_i^t \geq \sum_{j=1}^{n} v_j^t, \qquad \forall\ t \in T \tag{7}$$

Since it is assumed that depots do not request any empty containers, depots can be viewed as both dummy supply and demand nodes. This assumption allows us to manipulate the two commodity substitution problem, presented in the next subsection, as a balanced transportation problem.

2.3.2 Two-commodity substitution problem (Problem P2)

When we focus exclusively on the length of containers (i.e., $20'$ and $40'$), the substitution problem can be considered as a two-commodity transportation problem. The two-commodity substitution problem is presented below and is referred to as problem **P2** hereafter.

Problem **P2:**

$$\text{Minimize:} \qquad \sum_{t\in T}\sum_{t'\in T}\sum_{i=1}^{m+p}\sum_{j=1}^{n+p} c_{ij}^{tt'} x_{ij}^{tt'} \tag{8}$$

$$\text{Subject to} \qquad \sum_{j=1}^{n+p} x_{ij}^{tt} + \sum_{j=1}^{n+p} x_{ij}^{tt'} = u_i^t, \qquad \forall\ i \in I \bigcup P,\ t, t' \in T,\ t' \neq t \tag{9}$$

$$\sum_{i=1}^{m+p} x_{ij}^{tt'} - \sum_{k=1}^{\bar{n}} s_k^{tt'} = 0, \qquad \forall\ j \in J,\ t, t' \in T,\ t' \neq t \tag{10}$$

$$\sum_{i=1}^{m+p} x_{ij}^{tt} - \sum_{k=1}^{\bar{n}} s_k^{tt} = v_j^t, \qquad \forall\ j \in J,\ t \in T \tag{11}$$

$$\sum_{t\in T}\sum_{t'\in T} r_k^{tt'} s_k^{tt'} = e_k, \qquad \forall\ k \in \bar{J} \tag{12}$$

$$\begin{aligned} &x_{ij}^{tt'} \geq 0,\ \text{integer}, && \forall\ i,\ j,\ t,\ t' \\ &s_k^{tt'} \geq 0, && \forall\ k,\ t,\ t' \end{aligned} \tag{13}$$

where

T : the set of container types having two distinct lengths, $T = \{\ t_1,\ t_2 \}$.

$\bar{J}$: the subset of shippers that allow substitution, $\bar{J} \subseteq J$ and $|\bar{J}| = \bar{n}$.

$x_{ij}^{tt'}$: the number of type $t \in T$ containers transported from $i \in I \bigcup P$ to $j \in J \bigcup P$, which is to satisfy demands for type $t' \in T$ containers.

$c_{ij}^{tt'}$: the cost of transporting a type $t \in T$ container from $i \in I \bigcup P$ to $j \in J \bigcup P$, which is to satisfy demands for type $t' \in T$ containers.

$r_k^{tt'}$: the *substitution coefficient* (with respect to a 40′ container) between container types $t \in T$ and $t' \in T$ at demand node $k \in \bar{J}$. The variable $1/r_k^{t_i t_j}$ represents the number of containers (based on a 40′ container) of type t_i that is used to satisfy a request for type t_j at demand node k .

e_k : the sum of *extra requests* in demand node $k \in \bar{J}$. The extra request is the request which is unspecified in type; i.e., either types of T can satisfy it.

$s_k^{tt'}$: the slack variable that represents the extra supply of type $t \in T$ containers transported to satisfy extra request originated from the request for $t' \in T$ at $k \in \bar{J}$.

In **P2**, constraints (9) specify that the total number of empties moved from each consignee is equal to the number of supply of empties at that location. The slack variables $s_k^{tt'}$ in constraints (10) represent the number of substituted containers. Constraints (11) and (12) together guarantee that the number of empties arrived at each shipper is the same as the demand of empties at that location. Constraints (12) specify the demands which are satisfied using substitution containers. Finally, constraints (13) are the integer constraints.

To represent the substitution constraints in **P2**, we introduce a set of four slack variables for each demand node that allows substitution. These variables are corresponding to inter-connecting flows between two single commodity transportation networks. After augmenting two sets of inter-connecting flow variables (x_{ij}^{12} and x_{ij}^{21}), the incident matrix is partitioned into four row blocks to eliminated inequalities caused by the introduction of the sets of four slack variables.

Dantzig and Thapa described the integer solution property of transportation problems in [10]. In the Simplex method, it follows that all the basic variables have integer values if all row and column sums of coefficients in the incident matrix are integers and the elements of the demand vector are also integers. However, due to the non-symmetric rules in **P2**, some row and column sums may be non-integers. Hence, an optimal solution to the relaxed problem does not always yield an integer solution. That is the Simplex method cannot guarantee the integrality for **P2**.

To find an integer solution to **P2**, we consider a branch-and-bound (BNB) based algorithm. It should be mentioned that the BNB algorithm finds the optimal integer solution at the cost of requiring a great deal of memory space and computational time. To ameliorate the running time of the BNB algorithm, a forced integer programming (forced IP) method, as an approximation algorithm, is proposed. It follows from the fact that fractional coefficients of the substitution rule constraints force the slack variables and, subsequently, the other decision variables to be fractional. Hence, the forced IP method modifies the slack variables to be integers by re-allocating the slack variables. It takes advantage of the problem structure where there is a relatively few number of fractional coefficients which might cause a slight perturbation over an optimal integer solution.

For the sake of notational simplicity, let us define the optimal slack vector for demand node $k \in \bar{J}$ as

$$\mathbf{s}_k = [s_k^{t_1 t_1},\ s_k^{t_2 t_1},\ s_k^{t_1 t_2},\ s_k^{t_2 t_2}]', \qquad k \in \bar{J},\ T = \{\ t_1,\ t_2\}$$

which consists of corresponding four slack variables from an optimal non-integer solution. By a simple adjustment, the optimal slack vector can be modified to be an integer vector which is referred to as the *modified slack vector* and is denoted by $\mathbf{w}_k$,

$$\mathbf{w}_k = [w_k^{t_1 t_1},\ w_k^{t_2 t_1},\ w_k^{t_1 t_2},\ w_k^{t_2 t_2}]'.$$

Hence, constraints (10) in **P2** can be written as

$$\mathbf{r}_k' \mathbf{s}_k = \mathbf{r}_k' \mathbf{w}_k = e_k \tag{14}$$

where $\mathbf{r}_k = [r_k^{t_1 t_1},\ r_k^{t_2 t_1},\ r_k^{t_1 t_2},\ r_k^{t_2 t_2}]'$ is comprised of the substitution coefficients.

Forced IP Procedure for P2

1. Construct a set of $\mathbf{s}_k$ for all $k \in \bar{J}$ and initialize it with $k = 1$.
2. If $k > \bar{n}$, then go to step 7.
3. If $\mathbf{s}_k - \lfloor \mathbf{s}_k \rfloor = \mathbf{0}$, then $k = k + 1$ and go to step 2.
4. Construct a set of modified slack vectors, $W_k = \{\mathbf{w}_k :\ \mathbf{r}_k' \mathbf{w}_k = e_k\}$
5. Find the best modified slack vector $\mathbf{w}_k^* = \min_i f_{W_k(i)}$.
6. Replace $\mathbf{s}_k$ with $\mathbf{w}_k^*$ and go to step 3.

7. Solve the relaxed version of **P2** (referred to as **P2R**) after removing slack variables.

Where in the above procedure, the *floor function* $\lfloor \cdot \rfloor$ returns the largest integer equal to or less than its argument and $f_{W_k(i)}$ represents the objective function values to problem **P2R** when $\mathbf{s}_k$ is replaced with i- th entity of W_k.

Since all the slack variables in step 7 are modified into integers, they can be removed from **P2** by adding their values to the corresponding elements of the demand vector. The maximum number of modified slack vectors for each $k \in \overline{J}$ is $2 \times C_2^4 = 12$ where C_k^n is the number of different combinations of size k from a set of size n. Each pair of variables in a modified slack vector, which has at least one fractional value, is modified by a simple adjustment,

$$\mathbf{w}_k = \mathbf{s}_k + \Delta_k ,$$

where Δ_k is determined by modifying the optimal fractional slack vector into an integer vector. For example, if $\mathbf{s}_k = [2,\ 1.75,\ 0.5,\ 3]'$ and $\mathbf{r}_k = [\ 1, 1, 0.5, 1]'$, then there could be two candidates $\Delta_k = [\ 0, \text{-}0.75, 1.5, 0]'$ and $\Delta_k = [\ 0, 0.25, \text{-}0.5, 0]'$ which results in vectors $\mathbf{w}_k = [\ 2, 1, 2, 3]'$ and $\mathbf{w}_k = [\ 2, 2, 0, 3]'$ and satisfying $\mathbf{r}_k'\mathbf{s}_k = \mathbf{r}_k'\mathbf{w}_k = e_k = 7$. Since several variables are fractional and they inherently have the second or third power of 0.5 in their decimal places, this approximation algorithm yields a suboptimal integer solution in a polynomial number of iterations.

2.3.3 Multi-commodity substitution problem (Problem P3)

The multi-commodity substitution model is represented in Figure 1. Similar to the inter-connecting flow variables appeared in problem **P2**, dummy sets of demand nodes are introduced to represent substitution flow variables. These spawned sets of demand nodes complicate the problem structure and deteriorate the running time of a solution procedure. However, it provides a scalable structure to deal with substitution between multiple commodity types.

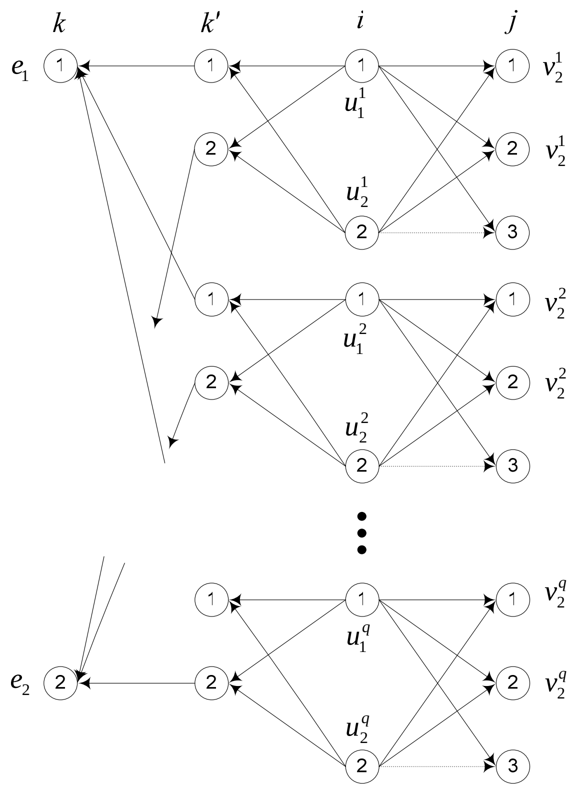


Figure 4: Multi-commodity substitution model, where $m=1$**,** $n=2$ **, and** $p=1$**.**

The multi-commodity substitution problem is presented below and is referred to as problem **P3**.

Problem **P3:**

$$\text{minimize} \quad \sum_{t\in T}\sum_{i=1}^{m+p}\sum_{j=1}^{n+p} c_{ij}^t x_{ij}^t + \sum_{t\in T}\sum_{i=1}^{m+p}\sum_{k=1}^{\bar{n}} c_{ik}^t x_{ik}^t \tag{15}$$

$$\text{subject to} \quad \sum_{j=1}^{n+p} x_{ij}^t + \sum_{k=1}^{\bar{n}} x_{ik}^t = u_i^t, \qquad \forall\ i \in I \cup P,\ t \in T \tag{16}$$

$$\sum_{i=1}^{m+p} x_{ij}^t = v_j^t, \qquad \forall\ j \in J,\ t \in T \tag{17}$$

$$\sum_{t\in T}\sum_{i=1}^{m+p} r_{ik}^t x_{ik}^t = e_k, \qquad \forall\ k \in \bar{J} \tag{18}$$

$$x_{ij}^t,\ x_{ik}^t \ge 0,\ \text{integer}, \qquad \forall\ i,\ j,\ k,\ t \tag{19}$$

where

T : the set of container types, $|T| = q$.

$\overline{J}$: the subset of shippers that allow substitution, $\overline{J} \subseteq J$ and $|\overline{J}| = \overline{n}$.

c_{ik}^t : the cost of transporting a type t container from supply node $i \in I \bigcup P$ to demand node $k \in \overline{J}$.

x_{ik}^t : the number of type t containers transported from supply node $i \in I \bigcup P$ to demand node $k \in \overline{J}$.

e_k : the sum of extra requests in demand node $k \in \overline{J}$, which is unspecified by a certain type.

Constraints (16) specify that the supply u_i^t can be shipped to satisfy both the exact type of requests from real nodes and the extra requests from dummy nodes. Constraints (17) indicate that the exact requests should be met. Finally, Constraints (18) indicates that the extra requests should be met without violating the substitution rule constraints. In this multi-commodity substitution problem, the overall demand on a demand node $k \in \overline{J}$ can be expressed as

$$v_k = \left(v_k^1;\ v_k^2;\ \cdots;\ v_k^q;\ e_k \right).$$

Therefore, the substitution coefficient r_{ik}^t in problem **P3** represents the number of container of type t to satisfy one extra request in demand node $k \in \overline{J}$.

Due to the introduction of the substitution mechanism which is allowed to distribute commodities without passing through the depots, the substitution variables augmented by the sets of dummy nodes are highly dependent to each other. However, the original decision variables (x_{ij}^t) are nearly independent to the substitution ones (x_{ik}^t). Therefore, after acquiring the optimal solution to the relaxed version of problem **P3** (**P3R**), problem **P3** can be decomposed into two problems. To do so, the decision variables of **P3** are divided into three sets.

1. the set of the original flow variables whose values are integer (non-fractional)

$$N = \left\{ (i, j, t):\ x_{ij}^{t*} = \text{integer} \right\} \qquad \forall\ i,\ j,\ t$$

2. the set of the original flow variables whose values are non-integer (fractional)

$$F = \left\{ (i, j, t):\ x_{ij}^{t*} = \text{non-integer} \right\} \qquad \forall\ i,\ j,\ t$$

3. the set of the substitution flow variables

$$S = \left\{ (i,k,t) : x_{ik}^{t\,*} \right\} \qquad \forall\ i,\ k,\ t$$

where $x_{ij}^{t\,*}$ and $x_{ik}^{t\,*}$ are the corresponding values in an optimal solution to problem **P3R**.

Accordingly, problem **P3** can be decomposed into two parts. Since only several out of several hundreds variables are fractional, it has less possibility to deviate from the optimal integer solution as long as it starts with the optimal relaxed solution. Using above defined notations, two sub-problems may be written as

Problem **P3F:**

$$\begin{aligned}
z_F = \text{ minimize } \quad & \sum_{(i,j,t)\in F} c_{ij}^t x_{ij}^t + \sum_{(i,k,t)\in S} c_{ik}^t x_{ik}^t \\
\text{subject to } \quad & \sum_{(i,j,t)\in F} x_{ij}^t + \sum_{(i,k,t)\in S} x_{ik}^t = \left(u_i^t\right)_F, \qquad && i \in I \cup P,\ t \in T \\
& \sum_{(i,j,t)\in F} x_{ij}^t = \left(v_j^t\right)_F, && j \in J,\ t \in T \\
& \sum_{(i,k,t)\in S} r_{ik}^t x_{ik}^t = e_k, && k \in \bar{J} \\
& x_{ij}^t,\ x_{ik}^t \ge 0, \text{ integer}, && \forall\ i, j, k, t.
\end{aligned}$$

Problem **P3N:**

$$\begin{aligned}
z_N = \text{ minimize } \quad & \sum_{(i,j,t)\in N} c_{ij}^t x_{ij}^t \\
\text{subject to } \quad & \sum_{(i,j,t)\in N} x_{ij}^t = \left(u_i^t\right)_N, \qquad && i \in I \cup P,\ t \in T \\
& \sum_{(i,j,t)\in N} x_{ij}^t = \left(v_j^t\right)_N, && j \in J,\ t \in T
\end{aligned}$$

where $\left(u_i^t\right)_F = u_i^t - \left(u_i^t\right)_N$ and $\left(v_j^t\right)_F = v_j^t - \left(v_j^t\right)_N$.

Decomposed IP Procedure for P3

1. Calculate an optimal solution to **P3R**.
2. Decompose into **P3F** and **P3N** and combine the set of non-integer variables.

   a) $y = \left\{ x_{ij}^t : (i,j,t) \in F \right\} \cup \left\{ x_{ik}^t : x_{ik}^t - \lfloor x_{ik}^t \rfloor \ne 0,\ (i,k,t) \in S \right\}$

3. If $y = \phi$, go to step 6.

4. Solve **P3F** by applying the BNB method only on y variables.

5. Update $y = \left\{ x_l : x_l - \lfloor x_l \rfloor \neq 0,\ x_l \in \mathbf{x}^*_{\text{P3F}} \right\}$ and go to step 3.

6. Combine the solution $\left\{ x^t_{ij} : (i, j, t) \in N \right\} \cup \left\{ x_l :\ x_l \in \mathbf{x}^*_{\text{P3F}} \right\}$, where $\mathbf{x}^*_{\text{P3F}}$ is an optimal solution to **P3F**.

As explained in the two commodity substitution problem, this decomposition algorithm yields a suboptimal integer solution in a polynomial number of iterations since several variables are fractional and they inherently have the second or third power of 0.5 in their decimal places.

2.3.4 Time complexity

For all solution methods presented here, the Simplex algorithm is repeatedly applied to solve the relaxed linear programming (LP) problem. This algorithm is defined by the pivot rule. This rule defines the way that decides which vertex of the polyhedron is selected when there are many basic feasible solutions (BFSs) to choose from [11]. Suppose that the pivot rule is always to move to the adjacent BFS which there is at lease increase in the objective function value. Under this pivoting rule, the Simplex algorithm requires $2^N - 1$ pivoting steps before terminating. More precisely, in a standard form, the time complexity of the LP is

$$O(MN2^N) = O(N2^N) = O(2^N),$$

where M is the number of rows and N is the number of columns of the incident matrix.

Let $\bar{n} = n$, then the time complexity of the relaxed LP is $O(2^{4(m+p)(n+q)+4n})$ for **P2** and $O(2^{q(m+p)(2n+p)})$ for **P3**. Due to the iterative employment of the LP, the complexity of the forced IP is $O(12n \cdot 2^{4(m+p)(n+q)+4n})$ for **P2**. Likewise, the complexity of the decomposed IP is $O(\alpha \cdot 2^{q(m+p)(\beta+n)})$ for **P3F**, where $2 \le \alpha < 20$ and $\beta \approx n + p$.

For the exact methods, unlike other pure IP problems, only slack variables or dummy variables need to be branched. In a worst case, since the branch-and-bound method could generate all leafs on the branch-and-bound tree and it performs the LP relaxation on every leaf, it takes exponential time $O(2^{4n} \cdot 2^{4(m+p)(n+q)+4n})$ for **P2** and $O(2^{q(m+p)n} \cdot 2^{q(m+p)(2n+p)})$ for **P3**. As described above, the Simplex algorithm has exponential time complexity. Its average behavior and worst case behavior have been studied and explained by Borgwardt [12] and Klee and Minty [13], respectively. There is no deterministic pivot rule under which the Simplex algorithm is known to take a sub-exponential number of iterations. However, the numerical behavior is in conflict with theoretical analysis [14]. That means that it is efficient in practice, while having no polynomial time worst-case complexity, although there are no satisfactory theoretical explanations of its excellent performance.

Therefore, for solving **P3**, the difference in the running time can be explained by the fact that the decomposed method is governed by the polynomial term α while the exact BNB method is governed by the exponential term $2^{q(m+p)n}$. Furthermore, as it might be expected, optimal solutions to a branched two sub-problems will be obtained relatively faster than its ancestor, unless they are non-convergent.

2.4 Simulation Experiments

In this section, we perform a series of simulation experiments to evaluate the developed approximation methods. The simulation experiments were coded in MATLAB ver. 7.0 (R14) with Optimization Toolbox ver. 3.0 and tested on a Pentium 2.53GHz PC.

2.4.1 Experiment 1: Two-commodity substitution

In this simulation experiment, the developed optimization method for the two-commodity substitution problem (**P2**), discussed in Subsection 2.3.2., is evaluated. Recall that the **P2** is about substituting two types of containers having two different dimensions. Here, we assume that the lengths of containers are $40'$ and $20'$. Without loss of generality, in this simulation experiment, we also assume that the total number of $40'$ containers is three times more than $20'$ containers and that among all the demands for the empties at shippers, 25% can be satisfied by using substitutable containers.

Table 2 shows the simulation results when the number of supply nodes m, and the demand nodes n are varied from 9 to 19 and 10 to 20, respectively. As seen, the number of depots is assumed to be one, p =1, and q=2 in indicates that the number of substitutable containers is two. The cost $c_{ij}^{tt'}$ in **P2,** which is the cost of transporting a type t container from node i to node j to satisfy demands for type t' container, is assumed to be the sum of the traveling time between nodes i and j and the empty handling time at nodes i and j. We assume that the cost $c_{ij}^{tt'}$ is randomly generated between 0 and 1 using a uniform random generator.

For each simulation scenario (i.e., each row) in Table 2, three different solution methods are applied and the results are compared. The first is the relaxed LP, in which integer constraints in Equation (11) are relaxed. The relaxed LP solution is found by applying the Simplex method. Obviously the optimal relaxed LP solution may not be feasible. The branch-and-bound (BNB) method, which uses bounds on the optimal cost to avoid exploring certain part of the feasible set, is used to find the optimal feasible integer solution. Finally, the developed forced IP method is used as an approximate solution method to find the solution to each simulation scenario.

Each simulation scenario in Table 2 is generated twenty times by randomly generating the cost $c_{ij}^{tt'}$ discussed above. For each scenario, the running time and the relative gap of the simulation results are found and averaged over 20 trials. The relative gap is defined as the difference in the objective function values of the integer solution and the relaxed LP solution.

Table 2: Performance of IP methods on P2 w.r.t. the number of nodes

m	n	p	q	relaxed LP	forced IP		exact BNB	
				time (sec)	time (sec)	gap (%)	time (sec)	gap (%)
9	10	1	2	0.7741	2.1666	0.0491	13.9641	0.0451
10	11	1	2	0.9725	2.7239	0.0391	10.8824	0.0188
11	12	1	2	1.2133	3.0345	0.0383	10.6219	0.0200
12	13	1	2	1.5532	3.8896	0.0234	19.9265	0.0178
13	14	1	2	2.2512	5.5008	0.0199	19.1534	0.0129
14	15	1	2	2.5958	6.6575	0.0221	25.5214	0.0175
15	16	1	2	3.1235	7.9711	0.0185	27.5277	0.0127
16	17	1	2	3.8726	10.9282	0.0247	51.0298	0.0086
17	18	1	2	4.6552	12.0875	0.0135	44.5260	0.0079
18	19	1	2	5.8399	14.3486	0.0199	68.9145	0.0096
19	20	1	2	6.7768	17.2153	0.0150	80.3998	0.0088

Table 2 indicates that the relative gap of the forced IP method is very close to that of the BNB method while the forced IP method is significantly faster than the BNB exact method. Note that the forced IP method takes the advantage of the problem structure where there is a relatively few number of fractional coefficients. For instance, for row 8 (i.e., m=15 and n=16) it can be observed that, in average, the forced IP method yields around 0.018% increase in the objective function value from the relaxed optimal solution and 0.006% increase from the exact BNB gap, while the time to generate the forced IP solution is about 3.5 times faster than that of the BNB solution.

2.4.2 Experiment 2: Multi-commodity substitution

In this simulation experiment, we assume that there are three container types (i.e., q=3) with different dimensions as given by (1). We use the decomposed IP method, which was developed in Section 2.3.3 as an approximation algorithm, to solve the multi-commodity substitution problem **(P3)**.

Table 3 shows the simulation results of the decomposed IP method to problem **P3** with three different container types. For each simulation scenario, 10 instances of problem are generated randomly, as described in Experiment 1, and the results of the 10 trials are averaged and shown in the table.

Table 3: Performance of IP methods on P3 w.r.t. the number of nodes

m	n	p	q	relaxed LP	decomposed IP		exact BNB	
				time (sec)	time (sec)	gap (%)	time (sec)	gap (%)
9	10	1	3	0.5030	0.9891	0.1287	14.2919	0.0373
10	11	1	3	0.7309	2.0214	0.1349	13.1345	0.0481
11	12	1	3	0.8842	2.9404	0.0995	22.6860	0.0161
12	13	1	3	1.1641	4.4844	0.0912	23.4359	0.0164
13	14	1	3	1.5296	4.0404	0.0781	23.4640	0.0210
14	15	1	3	1.7561	15.2653	0.1356	21.0688	0.0104
15	16	1	3	2.3610	8.8890	0.1220	81.7626	0.0174
16	17	1	3	2.7701	5.2608	0.0627	N/A	N/A
17	18	1	3	3.4971	11.0797	0.0690	N/A	N/A
18	19	1	3	4.3968	9.1123	0.0679	N/A	N/A
19	20	1	3	5.0031	10.2890	0.0464	N/A	N/A

N/A: the solution couldn’t be found

As indicated by Table 3, while the difference in objective function values between the decomposed IP and the exact method shows more than 0.1% difference in some cases, the decomposed IP method could yield an integer solution even if $m \geq 16$ and $n \geq 17$. In other words, while we faced the memory limitation in the BNB method as the number of consignees and shippers increase, the decomposed IP method constantly found the approximate integer solution with a little gap. Note that, in our simulation experiments, if any of the ten trials of each scenario had failed to yield a solution, the solution is marked unavailable (N/A).

Table 4 presents the sensitivity of the decomposed IP method with respect to the number of container types. In this simulation experiment, the container types are differentiated not only by their lengths and dimensions but also by their attributes, and the simulations were carried out on the network developed in [5]. The simulation network, which is developed for the Los Angeles/ Long beach port area, is explained later in Simulation Experiment 3 in detail. The cost is still assumed to be the sum of the traveling times between nodes and the container handling times at nodes. Table 4 shows the averaged results based on 10 independently generated trials.

Table 4: Performance of IP methods w.r.t. the number of types

m	n	p	q	relaxed LP	decomposed IP		exact BNB	
				time (sec)	time (sec)	gap (%)	time (sec)	gap (%)
12	8	3	5	1.5515	3.2641	0.0673	24.2282	0.0145
12	8	3	6	2.1016	8.2891	0.0496	83.4062	0.0092
12	8	3	7	2.7298	9.0843	0.0501	59.2800	0.0044
12	8	3	8	3.6547	5.6202	0.0277	126.8018	0.0072
12	8	3	9	4.5422	11.5796	0.0332	N/A	N/A
12	8	3	10	5.6922	11.7563	0.0132	N/A	N/A
12	8	3	11	6.4374	17.2186	0.0234	N/A	N/A
12	8	3	12	7.8875	14.7546	0.0069	N/A	N/A
12	8	3	13	8.8940	14.2740	0.0078	N/A	N/A
12	8	3	14	10.1999	30.4641	0.0102	N/A	N/A
12	8	3	15	11.5842	30.1839	0.0091	N/A	N/A

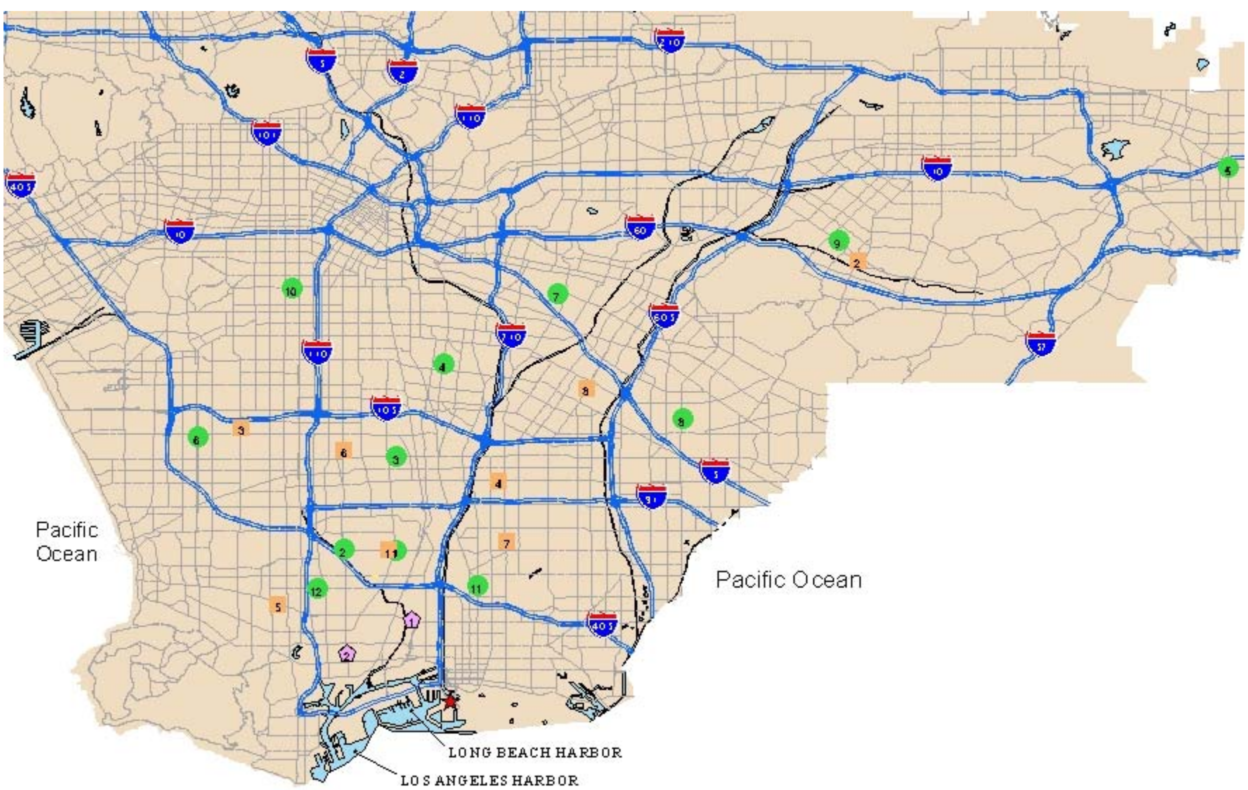
N/A: the solution couldn’t be found

As seen from Table 4, the maximum difference in objective function values between the decomposed IP and the exact methods is about 0.05%, which shows that the decomposed IP method was able to find a very good solution in significantly less amount of the time.

2.4.3 Experiment 3: Multi-commodity substitution in the LA/LB port area

In this simulation experiment, a case study is generated in the San Pedro Bay area located in Southern California containing the twin ports: Los Angeles and Long Beach. Current and Projected data are used to generate a scenario and to evaluate the costs associated with the empty container substitution methodologies developed in Section 2.

The geographical area shown in Figure 5 bounded from the West and South by the Pacific Ocean, from the East by freeway I-15, and from the North by freeway I-210 is considered for the case study simulation scenario. For the sake of simplicity, the area is referred to as the LA/LB port area.



The LA/LB port area, including the transportation network, was created in the ArcView Geographic Information System (GIS). On top of the ArcView GIS, we employed the "ArcView Network Analyst" to find the shortest distance and associated path between each pair of origin and destination (*OD*). In our study, origins are places where the empty containers are picked-up and destinations are those where empties are dropped-off. Shown in Figure 5 is the transportation network in the LA/LB port area, which consists of regional freeways, major avenues and main streets. Freeways I-110, and I-710 are the two arterial freeways carrying almost all containers into and out of LA/LB ports.

The case study consists of 12 consignees (supply of empties), 8 shippers (demand for empties), 2 local container depots, and one container terminal. The various symbols in Figure 5 represent the following entities:

- **Consignees:** The consignees are represented by circles in Figure 5 and labeled one to twelve. Many consignees were chosen to represent existing businesses. For instance, the first five locations (i.e., consignees 1 to 5) are the most active local importers in the LA/LB port area according to [1]. The other consignees' locations were distributed randomly throughout the region under study.

  The degree of activity among the consignees is variable. To each consignee *i* a weight w_{ci}, is associated which indicates its relative activity at that location compared to other consignees in the region. Vector W_C, below, shows the degree of activity at each consignee relative to other consignees.

$$W_C = [15 \quad 9 \quad 7 \quad 7 \quad 13 \quad 3 \quad 2 \quad 1 \quad 2 \quad 1 \quad 2 \quad 3] \qquad (20)$$

  For instance, vector W_C indicates that the degree of activity at Consignee 1 is 15 times more than that of Consignee 10. For the first five most active consignees in (20), the data were acquired from the terminal surveys conducted by Tioga Group [1]. Since the Tioga Group study does not include the low-active consignees, the degree of activity at consignees 6 to 12 was randomly assigned by generating a random number between 1 to 3.

Shippers: The shippers are represented by squares in Figure 5 and labeled one to eight. Likewise, to each shipper j a weight w_{sj}, is associated which indicates its relative activity at that location compared to other shippers. The degree of activity at shippers is shown in vector W_S below.

$$W_S = [5 \quad 4 \quad 1 \quad 2 \quad 1 \quad 1 \quad 1 \quad 2] \tag{21}$$

Similarly, the data of the most active exporters were acquired from the Tioga Group [1]. These data indicate that the first two locations are the most active local exporters in the LA/LB port area. For the other locations (i.e., shippers 3 to 8), random numbers were assigned.

It should be noted that the difference between the higher number of consignees and the lower number of shippers represents an imbalance between the number of import and export containers in the region.

- **Inland container depots**. The inland container depots are represented by pentagons in Figure 5 and labeled one to two. According to the Tioga Group[1], most existing container depots are located about 4 miles from the twin ports and 1 to 2 miles from freeways I-110 and I-710. The study identifies 10 depot locations in this area, among which we selected two without loss of generality. Depot 1 is the location of the Intermodal Container Terminal Facility (ICTF) located in the Los Angeles area.

- **Container terminal**. The container terminal's location is shown by a star in Figure 5 which represents the physical location of Pier G terminal which is one of the most active container terminals in the LA/LB port complex.

In this experiment, we generate a case study using current and projected data for the Los Angeles and Long Beach port area [1, 2].

According to [2], during 2000, almost 45% of trucks passing through the inbound gates of the LA/LB container terminals were delivering or picking up empties. Among all empties delivered to the LA/LB terminals, 24% were trucked from local shippers [1]. At the same time 49% of all empties picked up at terminals were destined for local consignees. In an extremely busy day in

2000, almost 5000 trucks were served at the inbound and outbound gates at a LA/LB terminal [2]. This translates to 267 empties trucked to local shippers and 547 empties delivered from local consignees.

Case Study **(2010 projection):** In this scenario, we use the projected figures for the empties demand and supply for 2010. It is expected that in 2010 the number of export and import loads in the LA/LB port area will be about 2.0 and 1.8 times more than those in the year 2000, respectively [1]. Thus, the number of empties demanded by shippers will be around 534, and the number of empties supplied by consignees will be about 985 containers per day in 2010. These figures are used as the case study.

We assume that the above total daily numbers of empties are distributed among the consignees and shippers according to their degree of activity presented in (20) and (21), respectively. We also assume that there are only three classes of containers t_1, t_2, and t_3, which are given by (2), in the LA/LB port area, and that the frequency of containers in each class is 50%, 25%, and 25% of the total number of containers, respectively.

The number of empties from each class of containers demanded by each shipper is then assumed to be divided into two parts: the exact and substitutable requests. The exact requests are the requests for the exact types of containers which cannot be substituted by any other types. The substitutable requests are those which can be substituted by other empties and are chosen, with equal probability, from one of the following substitution rules:

- Requests should be satisfied by exactly one specified types.
- Requests could be satisfied by any type with the same dimension.
- Requests could be satisfied by any type.

Table 5 shows the results of the simulation when the ratio of the number of substitutable requests to the total number of requests at shippers is varied from 0% to 100%. When this rate is selected to be 0% the problem is, in fact, reduced to three single commodity problems (**P1s**), each for one type of containers. Since the exact BNB method is very slow in finding the solutions and experiences out-of-memory frequently, the method is not considered here.

Here, the cost c_{ij}^t is chosen to be the traveling distance between consignee $i \in I \bigcup P$ and shipper $j \in J \bigcup P$.

Table 5: Performance of IP methods in Experiment 3

substitutable requests ratio	multi-commodity substitution problem			
	relaxed LP		decomposition IP	
	cost [miles]	time [s]	cost [miles]	time [s]
0%	18535.8300	1.5120	D/N	D/N
20%	18104.5300	1.3820	18110.8300	46.7570
40%	18033.9400	1.1810	18035.6200	49.3010
60%	17971.3700	1.2420	17972.5900	44.6840
80%	17923.5650	1.3020	17929.5800	54.1280
100%	17918.7200	0.4010	17924.8400	16.4640

D/N: don't need to run the IP method since the LP has the integer solution

As reported in Jula et al. [5], [19], the empty container reuse scenario results in more than 50% reduction in the empty trips activity around the port and depots in the LA/LB port area, as compared to the base scenario, which represent the current practices (i..e. no street-turn exchange of empties). Since in [5], the authors considered only one class of containers, their findings correspond to the 0% substitution scenario in Table 5. As seen from Table 5, the total empty traveling costs decreases as the total substitutable requests increases at shippers. Therefore, the cost of empty container reuse can be further reduced by allowing the substitution between empty containers in the region. This can be translated into further reduction in the traffic and congestion around the port and therefore, further reduction in noise and emissions.

3 Empty Interchange in Stochastic Environments: Modeling and Optimization

As discussed in Section 2, when all the information about the demand and supply of empty containers is available a priori, the empty container interchange problem can be modeled as a deterministic transportation problem. In the real world, however, one does encounter many sources of uncertainty, which could be related to spatial (location), temporal, or quantitative aspects of the future demand and supply.

In this section, we investigate the stochastic empty container interchange problem in which the supply and demand of empties follow known probability distributions. These probability distributions can be easily extracted from available historical data. We assume that the traveling costs between demand and supply nodes are deterministic and that no substitution is allowed. That is, we consider the single commodity transportation problem with stochastic supply and demand of empties at consignees and shippers, respectively.

3.1 Empty Interchange with Stochastic Supply and Demand

We start this subsection by assuming that the supply of empties is deterministic and that the probability distributions of demands are known and consist of finite sets of scenarios. We will see that the single commodity empty interchange problem with stochastic demand of empties can be modeled as a two-stage stochastic program. Later, we will study the empty interchange problem with both stochastic supply and demand of empties. We will model the problem as a one-stage stochastic program, a special case of the two-stage stochastic program.

3.1.1 Empty interchange with stochastic demand

Let the triplet (Ω, F, P) be a probability space, where F is a collection of events, $\Omega \in F$ is an event (the set of all possible scenarios), and P is the probability measure. Let ω be an outcome (i.e. a scenario) of event Ω which is a random experiment. Given the cost vector $\mathbf{c}$, the modified problem **P1** with stochastic demands can be expressed in the matrix-vector form as

$$\text{Minimize:} \qquad \mathbf{c}^T\mathbf{x} \tag{22}$$

$$\text{Subject to} \qquad \mathbf{A}_U\mathbf{x} = \mathbf{s} \tag{23}$$

$$\mathbf{A}_L\mathbf{x} = \mathbf{d}(\omega) \tag{24}$$

$$\mathbf{x} \geq \mathbf{0} \tag{25}$$

where $\mathbf{s}$ is the supply vector, and $\mathbf{d}(\omega)$ is the stochastic demand vector of all scenarios ω.

It should be noted that for a given decision vector $\mathbf{x}$ and a realization ω, constraints (24) should be met. To compensate for any constraints violation, we will provide a recourse vector **y,** that after observing the realization of ω, will affect the choice of $\mathbf{x}$. In other words, since the decision vector $\mathbf{x}$ in (22) must be made before the realization of ω is known, a second-stage linear program is introduced, whose values are uncertain but will influence the choice of $\mathbf{x}$. Therefore, the stochastic form of problem **P1** is modeled as a two-stage stochastic program

$$\begin{array}{llll} \min & \mathbf{c}^T\mathbf{x} + \mathbf{q}_+^T\mathbf{y}_+ + \mathbf{q}_-^T\mathbf{y}_- & & \\ s.t. & \mathbf{A}_U\mathbf{x} & & = \mathbf{s} \\ & \mathbf{A}_L\mathbf{x} & + \mathbf{y}_+ \quad - \mathbf{y}_- & = \mathbf{d}(\omega) \\ & & \mathbf{y}_+, \mathbf{y}_-, \mathbf{x} \geq \mathbf{0} & \end{array}, \tag{26}$$

where exogenous variables $\mathbf{y}_+$ and $\mathbf{y}_-$ are the second stage vectors, and $\mathbf{q}_+$ and $\mathbf{q}_-$ are the recourse cost vectors. The vector $\mathbf{q}^T = \begin{bmatrix} \mathbf{q}_+^T & \mathbf{q}_-^T \end{bmatrix}$ is introduced to penalize constraints violations, therefore it is required that $\mathbf{q}^T \geq 0$.

Since every scenario may involve a different set of constraints, a more reasonable objective is to choose the decision variables so that the expected cost of the following recourse formulation is minimized.

$$
\begin{aligned}
\min\ & \mathbf{c}^T\mathbf{x} + E_\omega[\mathbf{q}_+^T\mathbf{y}_+(\omega) + \mathbf{q}_-^T\mathbf{y}_-(\omega)] \\
s.t.\ & \mathbf{A}_U\mathbf{x} = \mathbf{s} \\
& \mathbf{A}_L\mathbf{x} + \mathbf{y}_+(\omega) - \mathbf{y}_-(\omega) = \mathbf{d}(\omega) \qquad \omega \in \Omega \\
& \mathbf{y}_+,\ \mathbf{y}_-,\ \mathbf{x} \geq \mathbf{0}
\end{aligned} \tag{27}
$$

where E_ω stands for the mathematical expectation, that is the weighted average over all ω. For notational simplicity, the model (27) is rewritten as

$$
\begin{aligned}
\min\ & \mathbf{c}^T\mathbf{x} + E_\omega[\mathbf{q}^T\mathbf{y}] \\
s.t.\ & \mathbf{A}_U\mathbf{x} = \mathbf{s} \\
& \mathbf{A}_L\mathbf{x} + \mathbf{W}\mathbf{y} = \mathbf{d}(\omega) \qquad \omega \in \Omega \\
& \mathbf{x},\ \mathbf{y} \geq \mathbf{0}
\end{aligned} \tag{28}
$$

where $\mathbf{W} = \left[\mathbf{I}_m \ \ -\mathbf{I}_m\right]$ is the recourse matrix; $\mathbf{I}_m$ is an identity matrix, and m is the number of shippers.

3.1.2 Empty interchange with stochastic supply

When both supply and demand of empties are stochastic, the stochastic empty interchange problem can be expressed by

$$
\begin{aligned}
\min\ & \mathbf{c}^T\mathbf{x} + E_\omega\left[\mathbf{q}_1^T\mathbf{y}_1 + \mathbf{q}_2^T\mathbf{y}_2\right] \\
s.t.\ & \mathbf{A}_U\mathbf{x} + \mathbf{W}_1\mathbf{y}_1 = \mathbf{s}(\omega) \qquad \omega \in \Omega \\
& \mathbf{A}_L\mathbf{x} + \mathbf{W}_2\mathbf{y}_2 = \mathbf{d}(\omega) \qquad \omega \in \Omega \\
& \mathbf{x},\ \mathbf{y}_1,\ \mathbf{y}_2 \geq \mathbf{0}
\end{aligned} \tag{29}
$$

where $\mathbf{W}_1 = \left[\mathbf{I}_n \ \ -\mathbf{I}_n\right]$ and $\mathbf{W}_2 = \left[\mathbf{I}_m \ \ -\mathbf{I}_m\right]$. n and m are the number of consignees and shippers, respectively. The stochastic problem in (29) is a one-stage stochastic program, hence, a subset of the two-stage stochastic program in (28). Therefore, in the rest of this report we will only consider and investigate the empty interchange problem with stochastic demand of empties.

3.2 Expected Value of the Stochastic Transportation Problem

In subsection 3.1.1 we modeled the stochastic transportation problem **P1** as a two-stage stochastic program. Using the deterministic equivalent, the two-stage stochastic program can be solved by linear programming techniques. For instance, suppose that Ω is a small set such that $|\Omega| = 2$ and $\omega_1, \omega_2 \in \Omega$. Suppose that p_1 and p_2 are the probability of ω_1 and ω_2, recursively, such that $p_1 + p_2 = 1$. Then, the deterministic equivalent of the model in (28) can be expressed as

$$\begin{array}{lllll} \min & \mathbf{c}^T\mathbf{x} + p_1\mathbf{q}^T\mathbf{y}^1 + p_2\mathbf{q}^T\mathbf{y}^2 & & & \\ s.t. & \mathbf{A}_U\mathbf{x} & & & = \mathbf{s} \\ & \mathbf{A}_L\mathbf{x} & + \mathbf{W}\mathbf{y}^1 & & = \mathbf{d}^1 \\ & \mathbf{A}_L\mathbf{x} & & + \mathbf{W}\mathbf{y}^2 & = \mathbf{d}^2 \\ & & & \mathbf{x},\ \mathbf{y}^1,\ \mathbf{y}^2 & \geq \mathbf{0} \end{array}, \tag{30}$$

where the superscripts $k = 1,2$ for demand vectors $\mathbf{d}^k$ and exogenous vector $\mathbf{y}^k$ are the scenario index corresponding to scenario ω_k. It should be noted that even for a moderate number of possible scenarios, the deterministic equivalent could result in a huge linear programming problem. For example, in the case of the stochastic transportation problem **P1,** we have assumed demands in the form of finite sets of scenarios. Since **P1** includes a set of depots which can be considered as super sources, every realization scenario ($\omega \in \Omega$) will be feasible, and hence, the number of feasible scenarios will be

$$|\Omega| = \prod_{j=1}^{n} |d_j(\omega)| \tag{31}$$

where n is the number of shippers, $d_j(\omega) = \{d_{j1}, \cdots, d_{js_j}\}$ is the finite set of demands (scenarios) at shipper j, and s_j is the cardinality of the set $d_j(\omega)$.

In other words, although Ω is a finite set, the number of realizable scenarios is too many to enumerate all. One way to overcome this problem is to use decomposition methods, such as Bender's decomposition method [15]. Unfortunately, although a decomposition method could reduce the number of variables substantially, the method still generates extremely large number of constraints [15]. For this reason, in this report, we use a sampling method technique called Monte Carlo simulation method to estimate the expected value of the stochastic program [16]

3.2.1 Monte Carlo simulation method

Consider the stochastic transportation problem **P1** in (28). Let v^* be the optimal expected value of (28), which can be expressed as the following two-stage stochastic program.

$$v^* = \min_{x \in S} \left\{ f(\mathbf{x}) \equiv E_\omega \left[g\left(\mathbf{x}, \mathbf{d}(\omega)\right) \right] \right\} \tag{32}$$

where $S \equiv \left\{ \mathbf{x} \middle| \mathbf{A}_U \mathbf{x} = \mathbf{s},\ \mathbf{x} \geq 0 \right\}$ and $g\left(\mathbf{x}, \mathbf{d}(\omega)\right) \equiv \mathbf{c}^T \mathbf{x} + \min_{\mathbf{y} \geq 0} \left\{ \mathbf{q}^T \mathbf{y} \middle| \mathbf{W}\mathbf{y} = \mathbf{d}(\omega) - \mathbf{A}_L \mathbf{x} \right\}$.

Let random samples $\mathbf{d}^1, \cdots, \mathbf{d}^N$ be N realizations of the random vector $\mathbf{d}(\omega)$, and $\hat{f}_N(\mathbf{x}) \equiv N^{-1} \sum_{k=1}^{N} g(\mathbf{x}, \mathbf{d}^k)$ be the sample average approximation (SAA) of $f(\mathbf{x})$. By replacing $f(\mathbf{x})$ with $\hat{f}_N(\mathbf{x})$ in (32), we find the optimal expected value of the *approximated* stochastic problem by

$$\hat{v}_N = \min_{x \in S} \left\{ \hat{f}_N(\mathbf{x}) \equiv N^{-1} \sum_{k=1}^{N} g(\mathbf{x}, \mathbf{d}^k) \right\} \tag{33}$$

Since the random realizations $\mathbf{d}^1, \cdots, \mathbf{d}^N$ have the same probability distribution as $\mathbf{d}(\omega)$, it follows that $\hat{f}_N(\mathbf{x})$ is an unbiased estimator of $f(\mathbf{x})$ for any $\mathbf{x}$ [17]. By generating M independent sample sets $\mathbf{D}^j = \left\{ \mathbf{d}^{1,j}, \cdots, \mathbf{d}^{N,j} \right\}$, j=1,…,M, each of size N, and solving the corresponding SAA problems in (33), the sample average of the optimal values of the approximated stochastic programs can be computed by

$$L_{N.M} = M^{-1} \sum_{j=1}^{M} \hat{v}_N^j \tag{34}$$

where $\hat{v}_N^j$ is the optimal value of SAA problem in (33) for each sample set $\mathbf{D}^j$. It can be shown that $L_{N.M} = M^{-1}\sum_{j=1}^{M}\hat{v}_N^j$ is an unbiased estimator of $E\left[\hat{v}_N\right]$ [17].

In stochastic optimization problems, the value of the stochastic solution (VSS) is defined as the difference between the optimal values of the stochastic problem and the deterministic problem computed by replacing stochastic variables by their mathematical expectations. The former is called the stochastic solution (SS) which is the solution of (32) and the latter is called the expected value (EV) solution. VSS indicates the benefit of knowing the distributions of the stochastic variables [18]

In particular, EV can be computed by taking the following procedure. During the first stage, a super-model solution is computed using the expected demand. Subsequently, with the first stage values fixed, each sub-model solution is independently computed and averaged over all sampled scenarios.

Let $\overline{\mathbf{x}}$ be the solution to the supermodel which is constructed by replacing random variables by their expectations. Hence, the expected value (EV) is obtained by

$$v_E = \min_{x\in S}\left\{ f(\mathbf{x}) = g\left(\mathbf{x}, E_\omega\left[\mathbf{d}(\omega)\right]\right)\right\} = g\left(\overline{\mathbf{x}}, \mu\right) \tag{35}$$

where $\mu = E_\omega\left[\mathbf{d}(\omega)\right]$.

We generate N independent random samples $\mathbf{d}^1,\cdots,\mathbf{d}^N$ of $\mathbf{d}(\omega)$. For each $\mathbf{d}^k$, k=1,…,N, we compute

$$v^k = g\left(\overline{\mathbf{x}}, \mathbf{d}^k\right) \tag{36}$$

where $g\left(\overline{\mathbf{x}}, \mathbf{d}^k\right) \equiv \mathbf{c}^T\overline{\mathbf{x}} + \min_{\mathbf{y}\geq 0}\left\{\mathbf{q}^T\mathbf{y} \,\middle|\, \mathbf{W}\mathbf{y} = \mathbf{d}^k - \mathbf{A}_L\overline{\mathbf{x}}\right\}$. The expectation of the EV (EEV) can be estimated by obtaining the sample average of $v^k = g\left(\overline{\mathbf{x}}, \mathbf{d}^k\right)$ over all the sampled scenarios, i.e.,

$$E\left[\hat{v}_E\right] = N^{-1}\sum_{k=1}^{N} g\left(\overline{\mathbf{x}}, \mathbf{d}^k\right) \tag{37}$$

Usually, if the difference

$$EEV - EV = E\left[\hat{v}_E\right] - v_E \tag{38}$$

is small, EEV is said to be a reasonably good solution to the stochastic program [18]. Furthermore, the value of the approximated stochastic solution (VSS) is computed by

$$VSS = EEV - ESS = E\left[\hat{v}_E\right] - E\left[\hat{v}_N\right] \tag{39}$$

This value of VSS in (39) indicates the price of using naive EV model instead of SS.

3.3 Simulation Experiment

In subsection 3.2, we modeled the empty container allocation problem with stochastic demand as a two-stage stochastic program. Furthermore, we used the Monte Carlo sampling method to obtain an estimate of the stochastic solution. In this section, we perform a series of simulation experiments to evaluate the stochastic solution (SS) and the expected value solution (EV) of the stochastic empty interchange problem. We assume that the distributions of empty demands are known and that they consist of finite sets of scenarios. These stochastic demands include the last-minute empty requests and cancellations within the working day.

Consider the simulation experiment 2.4.3 developed for the LA/LB port area. We use the projected figures for the empties demand and supply for 2010, and assume that the total daily numbers of empties are distributed among the consignees and shippers according to their degree of activities given in (20) and (21), respectively. Let d_j be the number of empties needed at shipper j obtained by applying the aforementioned procedure. We assume that the actual number of empty requested at shipper j is a discrete random number given by

$$d_j(\omega) = d_j + \omega \tag{40}$$

where ω is assumed to have the discrete uniform distribution, which can take any number with equal probability from the integer set

$$I_n = \{0, 1, \ldots, n\}. \tag{41}$$

Table 6 shows the optimal value for the expected value solution (EV) for different demand sets generated from different I_n. The Monte Carlo sampling method discussed in (36) and (37) is used to determine the expectation of the EV (EEV) when N is varied from 100 to 10,000. The third column in Table 6 presents the worst-case solution which is sometimes referred to as a *fat solution*. The fat solution will be the feasible solution for all possible realizations of ω in (40). More precisely, we let $\omega = n$, where n is the maximum possible realization of ω and is given in (41). The worst-case scenario is, then, solved as a single-commodity deterministic transportation problem [5].

In Table 6, similar to simulation experiment 2.4.3, the cost of moving an empty between a consignee and a shipper is assumed to be the traveling distance between these two nodes, thus has a deterministic value.

Table 6: Expected value solution of the approximating stochastic program

Demand Set	N	worst-case [miles]	Expected value solution		$\frac{\text{EEV - EV}}{\text{EV}}$ [%]
			EV [miles]	EEV [miles]	
Set 1 $I_5 = \{0, \cdots, 5\}$	100	10614.81	10291.35	10514.43	2.1676
	200			10516.71	2.1898
	300			10518.57	2.2079
	1000			10519.66	2.2185
	10000			10515.41	2.1772
Set 2 $I_{10} = \{0, \cdots, 10\}$	100	10633.98	10240.45	10505.59	2.5891
	200			10505.38	2.5871
	300			10505.59	2.5891
	1000			10503.82	2.5719
	10000			10504.32	2.5767
Set 3 $I_{15} = \{0, \cdots, 15\}$	100	10727.65	10018.77	10460.43	4.4083
	200			10471.15	4.5153
	300			10469.01	4.4940
	1000			10466.62	4.4701
	10000			10469.62	4.5001

Table 6 indicates that the worst-case solution is very expensive, and that the difference between EEV and EV is fairly small about 2 to 5%. Usually, if the difference is small, the EEV is reasonably good approximation to the solution of the stochastic problem [18], we will discuss this issue later.

Table 6 also indicates that the difference between EEV and EV is not sensitive to the sampling size N. Therefore, even the smallest independent sample size ($N = 100$) can be considered sufficient to capture the stochastic behavior of the demand in our stochastic transportation problem.

In Table 7, we calculate the estimated stochastic solution (ESS) using the Monte Carlo sampling method in 3.2.1 for different demand sets generated from different I_n.The EES is compared with the EEV when the sampling size N is 100, 200, and 300.

Table 7: Stochastic solution of the approximating stochastic program

Demand Set	N	EEV [miles]	confidence interval [miles]		ESS [miles] (M=10)	VSS [miles]	$\frac{VSS}{ESS}$ [%]
			90%	95%			
Set 1 $I_5 = \{0, \cdots, 5\}$	100	10514.43	±16.45	±19.66	10503.62	10.81	0.1029
	200	10516.71	±11.84	±14.14	10508.21	8.50	0.0809
	300	10518.57	±9.62	±11.47	10512.62	5.95	0.0566
Set 2 $I_{10} = \{0, \cdots, 10\}$	100	10505.59	±20.35	±24.32	10489.42	16.27	0.1551
	200	10505.38	±14.46	±17.26	10499.50	5.88	0.0560
	300	10505.59	±10.92	±13.03	10497.46	8.13	0.0774
Set 3 $I_{15} = \{0, \cdots, 15\}$	100	10460.43	±34.56	±41.30	10453.63	6.80	0.0650
	200	10471.15	±24.35	±29.06	10454.39	16.76	0.1603
	300	10469.01	±21.38	±25.50	10452.77	16.24	0.1554

The iteration number M for samplings is chosen to be 10 so that the ESS does not pass the 90% confidence interval. Table 7 shows the value of the approximated stochastic solution (VSS) given by (39). The VSS is very small relative to the total travel miles (about 0.2%). Therefore, one can approximate, with relatively small error, the expected value solution (EEV) as the optimal solution that minimizes the expected cost of the stochastic program.

4 Summary and Conclusions

In this study, we addressed the operational issues regarding the empty container reuse. In particular, we investigated the multi-commodity empty substitution problem, in which one type of containers can be substituted with another container. We modeled the problem analytically and proposed optimization techniques to find near optimal solutions. In order to investigate the efficiency of the proposed optimization techniques we developed realistic simulation scenarios using past, current and projected data in the Los Angeles/Long Beach (LA/LB) port area. Compared to single commodity empty container reuse problem studied in [5], the results of the simulated scenarios demonstrate that the cost of empty container reuse can be further reduced by allowing the substitution between empty containers in the (LA/LB) port area. This can be translated into further reduction in the traffic and congestion around the port and, therefore, further reduction in noise and emissions. The results of the simulation studies also indicate that the total empty traveling costs decreases as the total substitutable requests increases at shippers.

In addition to multi commodity container reuse the effect of uncertainties associated with demand and supply requirements on our approach has also been investigated by studying the stochastic empty container interchange problem where the supply and demand of empties follow known probability distributions. In this case we modeled the problem as a stochastic problem with recourse and developed optimization techniques to minimize the cost of empty container interchange in stochastic environments. The results showed that our approach can also perform as well as in a stochastic environment.

5 Implementation

The empty container interchange is beneficial to all parties involved in empty movements. The results of this report demonstrate that the operational issue (i.e., empty container mismatch, lack of perfect information about the number of requests) can be overcome using the developed methods. The implementation of the proposed approach however requires additional efforts which involve overcoming barriers which may include policy, collaboration, competition, legal, insurance and other issues that are outside the scope of this study. The benefits of empty container reuse demonstrated in this report present a strong motivation for overcoming whatever barriers exist to make the proposed approach implementable.

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