The Robust Vehicle Routing Problem (RVRP)

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Motivation

In delivery/routing industry:

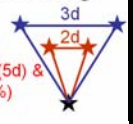
- the demand is uncertain
- the travel time is uncertain

We need efficient routing solutions taking into account the uncertainty!

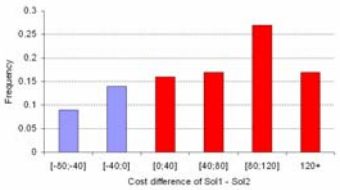
Ignoring Uncertainty in Routing

Solution 1 (Sol1): Cost: low mean (4d) & high variance (100%)

Solution 2 (Sol2): Cost: high mean (5d) & low variance (10%)



Simulation results over 100 realizations of uncertainty with d=100



Addressing Uncertainty

Life is uncertain, so is business!

Instead of rolling the dice, we can use:

- Stochastic Optimization
- Probabilistic Analysis
- Real-time Routing
- Robust Optimization

Robust Optimization

Immunize the uncertainty with respect to worst case scenario!

RVRP: find the best route for the worst case realization of uncertainty

- Scenario based uncertainty
- Uncertainty in Demand
- Uncertainty in Travel Time

Uncertainty Sets

A little bit of geometry…

- Convex Hull

$Y = \{y \mid y_k \geq 0, \sum_k y_k \leq 1, k = 1, \ldots, s\}$

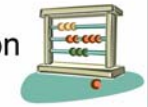
- Ellipsoidal

$Y = \{y \mid \sum_k y_k^2 \leq 1, k = 1, \ldots, s\}$

- Box

$Y = \{y \mid -1 \leq y_k \leq 1, k = 1, \ldots, s\}$

RVRP Formulation



A little bit of math…

min Total Travel Time

s. t. Routing constraints

MTZ constraints: $u_j - u_i + C(1 - x_{ij}) \geq D$

Adding uncertainty in Demand:

- Convex Hull: $D = d_j^0 + \max\{\max_k d_j^k, 0\}$
- Ellipsoidal: $D = d_j^0 + \sqrt{\sum_k (d_j^k)^2}$
- Box: $D = d_j^0 + \sum_k |d_j^k|$

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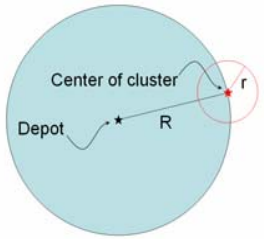
Clustering in Network

A measure of clustering: R/r ratio



Performance Measures

Relative extra cost of the robust solution

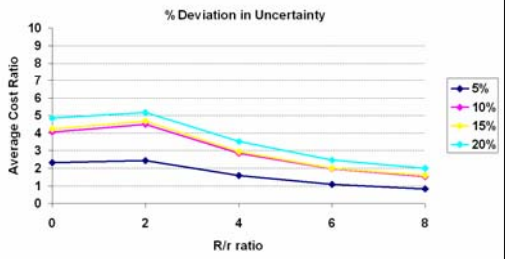
Cost Ratio: $r_{cd} = \frac{z_R - z_D}{z_D}$

Relative unsatisfied demand of the deterministic solution

Unmet Demand Ratio: $r_{ud} = \frac{z_{ud}}{\sum_{i \in V} d_i^0}$

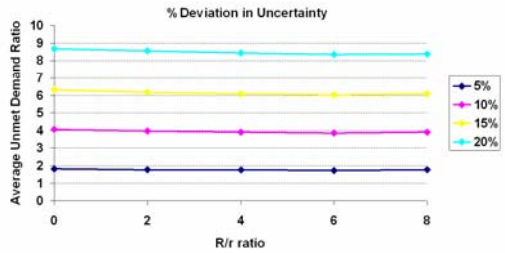
Experimental Results

Cost of Robust Solution

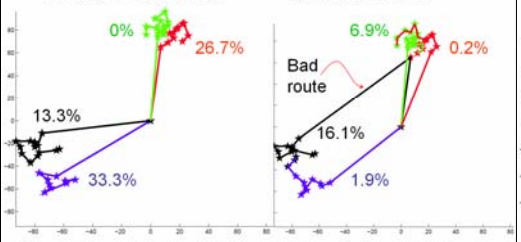


Experimental Results

Unmet Demand of Deterministic Solution



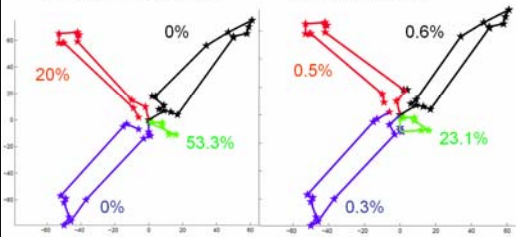
A Bad Case



Unmet Demand Ratio: 4.2%

Cost Ratio: 12.81%

A Good Case



Unmet Demand Ratio: 6.76%

Cost Ratio: 0.78%

Conclusions

Slack in the vehicles is the key!

A strategic distribution of slack over the clusters creates synergy between vehicles

Robust solution benefits from this synergy with little extra cost

Future Work

- In addition to uncertainty in demand, consider uncertainty in cost
- Develop a special solution procedure
- Implement a real-life application: UPS