Empty Container Reuse in the Los Angeles/Long Beach Port Area

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Abstract: In the Los Angeles/Long Beach (LA/LB) port complex, the empty containers are handled twice; once they are recycled from importers and the second time they are trucked to exporters. Clearly a system, which facilitates the interchange of empties outside the ports, will reduce the traffic congestion and emissions around the ports.

In this paper, the deterministic and stochastic empty container reuse problems are considered. The problems are modeled analytically, and approximation solution methods are developed. The developed optimization methods are evaluated using realistic simulation scenarios generated using past, current, and projected data from the LA/LB port area. Simulation results show the efficiency of the developed algorithms in terms of computational time and solution quality.

1. Introduction

The Los Angeles and Long Beach (LA/LB) port complex, located in San Pedro Bay, is the largest U.S. ocean freight hub and the busiest container port complex. It consists of fourteen individually gated terminals, and serves as a crucial node in the regional supply chain (Mallon and Magaddino, 2001). Over the last decade, the growth in container traffic in the LA/LB port was significant. It is estimated that the container traffic in 2020 will be around 28 million TEUs or almost 15.1 million containers. Thus, by 2020 the volume of containers moving through the combined LA/LB ports will be at least three times the current volume (Mallon and Magaddino, 2001).

As a consequence, port generated traffic has emerged as a major contributor to regional congestion. Traffic congestion and long queues at the gates of the LA/LB terminals are the main source of air pollution, wasted energy, driver inefficiency, and increasing maintenance cost imposed by the volume of trucks on the roadway (Barton, 2001). Moreover, a study by (California Highway Patrol, 2000) in southern California freeways reveals that the I-710 freeway topped the list of freeway collisions on two measures, the highest proportion of truck-involved collisions, 31%, and truck-caused collisions at 16%.

There are numerous ways to improve traffic congestion at ports. Options include developing new facilities and expanding current ones, deploying advanced technologies, and improving operational characteristics at ports. The scarcity of land at major ports, however, has made the option of developing new facilities, if not infeasible, significantly costly. Feasible options are therefore those that rely on more intelligent decision making to make current operations more efficient.

Figure 1 shows the empty container flows to and from the LA/LB container terminals, graphically. The loaded containers arriving at the port are picked up and transported by trucks to their destinations. After having been unloaded, the emptied containers are picked up and typically moved back to the port (arrow 5 - i.e., arrow labeled 5 in Figure 1). The exporters, who need empty containers to fill with exportable goods, will hire a trucking company to pick up the empties from the port and transport them to their locations (arrow 2). After empties have been loaded at the export firm, the truckers will transport them back to the port where they will be loaded on the ship for export.

It is clear that a system, which facilitates the interchange of empties outside the ports, is not only desirable but also necessary. The idea of empty container "reuse" consists of using emptied import containers for export loads without first returning them to marine terminals. Generally speaking, two major methodologies can be considered for reusing empty containers: depot-direct (arrows 8 and 0) and street-turn (arrow 9) (The Tioga Group, 2002, Jula *et al.*, 2003).

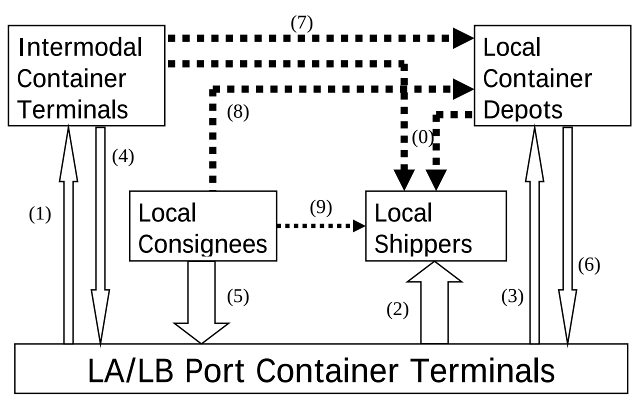

Figure 1. empty container flows including depot-direct and street-turn

Despite the importance of the empty container reuse problem, the research efforts in this area have been scant. As noted in (Dejax and Crainic, 1998), even the work on developing models related to the container transportation problems is very limited. (Crainic *et al.*,1993) proposed dynamic and stochastic models for empty container allocation in a land distribution and transportation system. (Cheung and Chen, 1998) formulated the dynamic container allocation problem as a two-stage stochastic network model. The model assists liner operators to allocate their empty containers and consequently reducing their leasing cost and the inventory level at ports. In another related work, (Choong *et al.*, 2002) addressed the effect of the length of the planning horizon on empty container management. They used the intermodal container-on-barge operation in Mississippi river as the case study to investigate the advantages of using a long rolling horizon.

The empty container allocation problem was also investigated by (Jula *et al.*, 2003 and 2006). The authors proposed a model and an optimization technique for the dynamic allocation of empty containers in the LA/LB port and its vicinity. They showed that the empty container reuse can yield a significant reduction in the number and cost of truck trips.

In this paper, based on the empty container interchange problem presented in (Jula *et al.*, 2006), we investigate the possibility of fulfilling the request of one type of containers with another type in an effort to further reduce the cost. This problem is referred to as the multi-commodity empty container substitution problem. In addition, this paper investigates the empty container interchange problem in stochastic environments.

2. SINGLE AND MULTI COMMODITY EMPTY CONTAINER INTERCHANGE

In this section, we first review container types and then propose models and optimization techniques for the empty container interchange problem in deterministic environments.

2.1 Container types

Containers can be classified into separable types (classes) according to their intended use, external dimension, ownership, etc. For instance, if the types of containers can be determined by only three attributes: purpose, dimensions, and ownership, a type t container can be expressed as

$$\text{type } t = \{ \text{ purpose, dimension, ownership } \}.$$

The purpose indicates the intended use of containers such as general (dry cargo) or specific purpose (refrigerated, specialized, etc.) containers. Most containers are sized according to the International Standards Organization (ISO). Based on ISO, containers are described in terms of TEU (Twenty-foot Equivalent Units) in order to facilitate comparison of one container system with another. A TEU is an 8 feet wide, 8 feet high and 20 feet long container. An FEU is an 8-foot high, 40-foot long container and is equivalent to two TEUs. Containers with height of 9.5 feet are usually referred to as high cube containers. The most widely used containers are general purpose FEU containers. Without loss of generality, in the rest of this paper, we will only consider standard dry cargo containers with the following standard dimensions:

$$D_1:\ 40' \times 8' \times 8.5',\quad D_2:\ 20' \times 8' \times 8.5',\quad D_3:\ 40' \times 8' \times 9.5' \qquad (1)$$

2.2 Substitution rules

The substitution rules are the rules specified for substituting each ordered pair of container types. These rules may be symmetric or asymmetric. For instance, suppose that there are three types of containers,

$$t_1 = \{ \text{ General, } \mathrm{D}_1\text{, Hanjin } \},\ t_2 = \{ \text{ General, } \mathrm{D}_2\text{, Hyundai } \}\text{, and } t_3 = \{ \text{ General, } \mathrm{D}_3\text{, Maersk } \}. \qquad (2)$$

The possible asymmetric container substitutions between t_1, t_2, and t_3 could be as follows.

Asymmetric in type for (t_1, t_3): One request for t_3 could be fulfilled by one supply of t_1, but the reverse substitution is not permitted. This case may happen when certain customers do not accept the high cube containers t_3 due to their facility limitation.

Asymmetric in number for (t_2, t_1): One request for t_1 can be fulfilled by two supplies of t_2. However, sometimes, two requests for t_2 can only be satisfied by two supplies of t_1. This case happens when export cargos in a shipper location have two different destinations or when it is desirable to have two small containers because of the weight limitation.

Generally speaking, the substitution rules come from differences in the handling capacity of the loading/unloading facilities, the destination of cargos, the weight of cargos, or the nature of cargos. Other factors may include operational regulations or limitations set forth by freight liners or trucking companies.

Let T be the set of container types and v_k^t be the number of requested empties of type $t \in T$ at shipper k. We assume that the request consists of (a) exact and (b) substitutable parts. The exact request for container type t at shipper k, denoted by $v_{k,e}^t$, is the request which must be fulfilled by the exact type t, whereas the substitutable request, denoted by $v_{k,s}^t$, can be fulfilled by any container types. Hence, we have

$$v_k^t = v_{k,e}^t + v_{k,s}^t .$$

We define the extra request at shipper k with respect to an FEU as

$$e_k = \sum_{t \in T} a^t v_{k,s}^t \tag{3}$$

where $1/a^t$ indicates the number of containers of type t needed to substitute one FEU container. For instance, if the extra request is for one container of type t_1, where t_1 is defined in (2), the request can be fulfilled by one t_1, or substituted by either two t_2 (i.e., $a^t = 0.5$) or one t_3 (i.e., $a^t = 1$).

2.3 Single commodity interchange problem

When there is no substitution allowed, this problem can be decomposed into a series of single commodity transportation problems for each empty container type. The single commodity transportation problem is presented below, which, hereafter, is referred to as problem **P1**.

Problem **P1:**

Minimize: $$\sum_{t\in T}\sum_{i=1}^{m+p}\sum_{j=1}^{n+p} c_{ij}^t x_{ij}^t$$

Subject to $$\sum_{j=1}^{n+p} x_{ij}^t = u_i^t, \qquad \forall\ i \in I \bigcup P,\ t \in T \qquad (4)$$

$$\sum_{i=1}^{m+p} x_{ij}^t = v_j^t, \qquad \forall\ j \in J,\ t \in T \qquad (5)$$

$$x_{ij}^t \geq 0,\ \text{integer}, \qquad \forall\ i \in I \bigcup P,\ j \in J,\ t \in T \qquad (6)$$

where

I : the set of consignees, $|I| = m$.

J : the set of shippers, $|J| = n$.

P : the set of depots including terminals, $|P| = p$.

T : the set of container types, $|T| = q$.

c_{ij}^t : the cost of transporting a type $t \in T$ container from supply node $i \in I \bigcup P$ to demand node $j \in J \bigcup P$.

x_{ij}^t : the decision variable that represents the number of type $t \in T$ containers transported from supply node $i \in I \bigcup P$ to demand node $j \in J \bigcup P$.

u_i^t : the number of available empties of type $t \in T$ in supply node $i \in I \bigcup P$.

v_j^t : the number of requested empties for type $t \in T$ in demand node $j \in J$.

In **P1**, constraints (4) ensure that the total number of empties moved from each consignee is equal to the number of supply of empties at that location. Constraints (5) guarantee that the number of empties arrived at each shipper is the same as the demand of empties at that location. Finally, constraints (6) are the integer constraints.

We assume that the total number of available containers of type t in supply nodes $I \bigcup P$ is greater than or equal to the total number of requested empties of type t in demand nodes J . In other words, we assume that all the demands can be fulfilled by internal supplies, rather than exogenous resources. Hence, we have

$$\sum_{i=1}^{m+p} u_i^t \geq \sum_{j=1}^{n} v_j^t, \qquad \forall\ t \in T$$

Since it is assumed that depots do not request any empty containers, depots can be viewed as both dummy supply and demand nodes.

2.4 Multi-commodity substitution problem

The multi-commodity substitution model is shown graphically in Figure 2. The node i and j represent the supply and demand nodes, respectively. The exact type of requests is assigned to the demand node j . The dummy demand node k' is introduced to represent substitutable requests. Also, the extra request generated by (3) is assigned to the extra request node k .

These spawned sets of dummy demand and extra request nodes complicate the problem structure and deteriorate the running time of a solution procedure. However, it provides a scalable structure to deal with substitution between multiple commodity types.

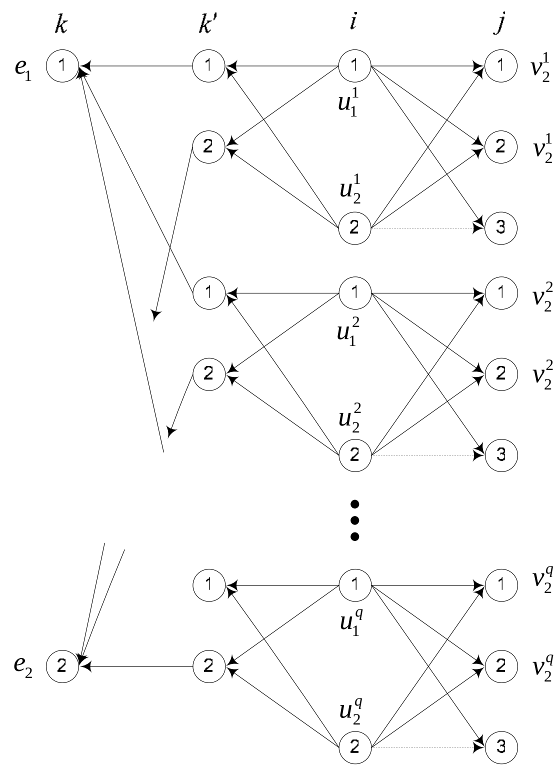

Figure 2. Multi-commodity substitution model, where $m=1$, $n=2$, and $p=1$.

The analytical model of the multi-commodity substitution problem is presented below, which hereafter is referred to as problem **P2**.

Problem **P2:**

$$\text{minimize} \quad \sum_{t\in T}\sum_{i=1}^{m+p}\sum_{j=1}^{n+p} c_{ij}^t x_{ij}^t + \sum_{t\in T}\sum_{i=1}^{m+p}\sum_{k=1}^{\bar{n}} c_{ik}^t x_{ik}^t \tag{8}$$

$$\text{subject to} \quad \sum_{j=1}^{n+p} x_{ij}^t + \sum_{k=1}^{\bar{n}} x_{ik}^t = u_i^t, \qquad \forall\ i \in I \cup P,\ t \in T \tag{9}$$

$$\sum_{i=1}^{m+p} x_{ij}^t = v_j^t, \qquad \forall\ j \in J,\ t \in T \tag{10}$$

$$\sum_{t\in T}\sum_{i=1}^{m+p} r_{ik}^{t} x_{ik}^{t} = e_k, \qquad \forall\ k \in \bar{J} \tag{11}$$

$$x_{ij}^{t},\ x_{ik}^{t} \geq 0,\ \text{integer}, \qquad \forall\ i,\ j,\ k,\ t \tag{12}$$

where

T : the set of container types, $|T| = q$.

$\bar{J}$: the subset of shippers that allow substitution, $\bar{J} \subseteq J$ and $|\bar{J}| = \bar{n}$.

c_{ik}^{t} : the cost of transporting a type t container from supply node $i \in I \bigcup P$ to demand node $k \in \bar{J}$.

x_{ik}^{t} : the number of type t containers transported from supply node $i \in I \bigcup P$ to demand node $k \in \bar{J}$.

e_k : the sum of extra requests in demand node $k \in \bar{J}$, which is unspecified by a certain type.

$r_k^{t_i t_j}$ is the substitution rule coefficient between container type t_i and t_j at demand node k . That is, $1/r_k^{t_i t_j}$ indicates the number of containers of type t_i needed to satisfy one extra request originated from the requests for t_j at k .

Constraints (9) specify that the supply u_i^t can be shipped to satisfy both the exact requests at actual nodes and/or extra requests at dummy nodes. Constraints (10) indicate that the exact requests should be met. Finally, Constraints (11) indicates that the extra requests should be met without violating the substitution rule constraints. In this multi-commodity substitution problem, the overall demand at a demand node $k \in \bar{J}$ can be expressed as

$$v_k = \left(v_k^1;\ v_k^2;\ \cdots;\ v_k^q;\ e_k\right).$$

Therefore, the substitution coefficient r_{ik}^{t} in problem **P2** represents the number of container of type t to satisfy one extra request in demand node $k \in \bar{J}$.

According to the integer solution property of transportation problem in (Dantzig and Thapa, 1997), the optimal solution to the relaxed version of the problem (**P2R**) does not always yield an integer solution. That is the Simplex method cannot guarantee the integrality for **P2**. So, we consider a branch-and-bound (BNB) algorithm to find an integer solution. It should be mentioned that the BNB algorithm finds the optimal solution at the cost of requiring a great deal of memory space and computational time. To ameliorate the running time the BNB, an approximation method is proposed.

Due to the introduction of the substitution mechanism which is allowed to distribute commodities without passing through the depots, the substitution variables augmented by the sets of dummy nodes are highly dependent to each other. However, the original decision variables (x_{ij}^{t}) are nearly independent to the substitution ones (x_{ik}^{t}). Therefore, after acquiring the optimal solution to **P2R**, **P2** can be decomposed into two problems. To do so, the decision variables of **P2** are divided into three sets.

1) The set of the original flow variables whose values are integer (non-fractional)

$$N = \left\{(i, j, t):\ x_{ij}^{t\,*} = \text{integer}\right\} \qquad \forall\ i,\ j,\ t$$

2) The set of the original flow variables whose values are non-integer (fractional)

$$F = \left\{(i, j, t):\ x_{ij}^{t\,*} = \text{non-integer}\right\} \qquad \forall\ i,\ j,\ t$$

3) The set of the substitution flow variables

$$S = \left\{(i, k, t):\ x_{ik}^{t\,*}\right\} \qquad \forall\ i,\ k,\ t$$

where $x_{ij}^{t\,*}$ and $x_{ik}^{t\,*}$ are the corresponding values in an optimal solution to **P2R**.

Accordingly, **P2** can be decomposed into two parts. Since only several out of several hundreds variables are fractional, it has less possibility to deviate from the optimal integer solution as long as it starts with the optimal relaxed solution. Using above defined notations, two sub-problems may be written as

Problem **P2F:**

$$
\begin{aligned}
z_F = \text{ minimize } \quad & \sum_{(i,j,t)\in F} c_{ij}^t x_{ij}^t + \sum_{(i,k,t)\in S} c_{ik}^t x_{ik}^t \\
\text{subject to } \quad & \sum_{(i,j,t)\in F} x_{ij}^t + \sum_{(i,k,t)\in S} x_{ik}^t = \left(u_i^t\right)_F, && i \in I \bigcup P,\ t \in T \\
& \sum_{(i,j,t)\in F} x_{ij}^t = \left(v_j^t\right)_F, && j \in J,\ t \in T \\
& \sum_{(i,k,t)\in S} r_{ik}^t x_{ik}^t = e_k, && k \in \overline{J} \\
& x_{ij}^t,\ x_{ik}^t \geq 0, \text{ integer}, && \forall\ i,j,k,t.
\end{aligned}
$$

Problem **P2N:**

$$
\begin{aligned}
z_N = \text{ minimize } \quad & \sum_{(i,j,t)\in N} c_{ij}^t x_{ij}^t \\
\text{subject to } \quad & \sum_{(i,j,t)\in N} x_{ij}^t = \left(u_i^t\right)_N, && i \in I \bigcup P,\ t \in T \\
& \sum_{(i,j,t)\in N} x_{ij}^t = \left(v_j^t\right)_N, && j \in J,\ t \in T
\end{aligned}
$$

where $\left(u_i^t\right)_F = u_i^t - \left(u_i^t\right)_N$ and $\left(v_j^t\right)_F = v_j^t - \left(v_j^t\right)_N$.

P2N consists of the variables of the set N, which already has an integer solution. And, P2F consists of the variables of the set F and S. Since the substitution rule coefficients disturbed the entire solution to be fractional, the variables of the set S is assigned to P2F as candidates to be branch and bounded.

Decomposed IP Procedure for P2

1. Calculate an optimal solution to **P2R**.

2. Decompose into **P2F** and **P2N** and combine the set of non-integer variables.

$y = \left\{ x_{ij}^{t} : (i, j, t) \in F \right\} \cup \left\{ x_{ik}^{t} : x_{ik}^{t} - \left\lfloor x_{ik}^{t} \right\rfloor \neq 0, \ (i, k, t) \in S \right\}$

3. If $y = \phi$, go to step 6.

4. Solve **P2F** by applying the BNB method only on y variables.

5. Update $y = \left\{ x_l : x_l - \left\lfloor x_l \right\rfloor \neq 0, \ x_l \in \mathbf{x}_{\mathrm{P3F}}^{*} \right\}$ and go to step 3.

6. Combine the solution $\left\{ x_{ij}^{t} : (i, j, t) \in N \right\} \cup \left\{ x_l : x_l \in \mathbf{x}_{\mathrm{P3F}}^{*} \right\}$, where $\mathbf{x}_{\mathrm{P3F}}^{*}$ is an optimal solution to **P2F**.

It follows that the fractional coefficients of the substitution rule coefficients force to substitution variables and, subsequently, the other decision variables to be fractional. Hence, this approximation method applies a BNB algorithm to only some substitution variables which are fractional. Since several variables out of several hundreds are fractional and they inherently have the second or third power of 0.5 in their decimal places, the decomposition algorithm yields a suboptimal integer solution in a polynomial number of iterations.

2.5 Simulation Experiments

In this section, we perform a series of simulation experiments to evaluate the developed approximation methods in Subsections 2.3 and 2.4. The algorithms were coded in MATLAB ver. 7.0 (R14) with Optimization Toolbox ver. 3.0 and tested on a Pentium 2.53GHz PC.

2.5.1 Simulation Experiment 1 (a random scenario)

In this scenario, we randomly distribute supply and demand nodes in a Euclidean plane. The cost is assumed to be the sum of the traveling time between each supply and demand nodes as well as the handling time at those nodes. We randomly generated this cost between 0 and 1 using a uniform random generator.

Three different solution methods are applied and compared. The first is the relaxed LP, in which integer constraints are relaxed. The relaxed LP solution is found by applying the Simplex method. Obviously the optimal relaxed LP solution may not always be feasible. The branch-and-bound (BNB) method, which uses bounds on the optimal cost to avoid exploring certain part of the feasible set, is used to find the optimal feasible integer solution. Finally, the decomposed IP method is used as an approximate solution method to find the suboptimal integer solution.

Table 1 shows the averaged results over 10 independently generated trials. It represents the sensitivity of the decomposed IP method with respect to the number of container types. Note that the container types are differentiated not only by their lengths and dimensions but also by their attributes.

The relative gap, in Table 1, is defined as the difference in the objective function values of the integer solution and the relaxed LP solution. It should be noted that the BNB method is frequently faced with memory limitation when the number of container types is increased. In Table 1, if any of the ten trials fails to yield a solution, the solution is marked not-available (N/A).

Table 1. Performance of IP methods w.r.t. the number of types

m	n	p	q	Relaxed LP	Decomposed IP		Exact BNB	
				time (sec)	time (sec)	gap (%)	time (sec)	gap (%)
12	8	3	5	1.5515	3.2641	0.0673	24.2282	0.0145
12	8	3	6	2.1016	8.2891	0.0496	83.4062	0.0092
12	8	3	7	2.7298	9.0843	0.0501	59.2800	0.0044
12	8	3	8	3.6547	5.6202	0.0277	126.8018	0.0072
12	8	3	9	4.5422	11.5796	0.0332	N/A	N/A
12	8	3	10	5.6922	11.7563	0.0132	N/A	N/A
12	8	3	11	6.4374	17.2186	0.0234	N/A	N/A
12	8	3	12	7.8875	14.7546	0.0069	N/A	N/A
12	8	3	13	8.8940	14.2740	0.0078	N/A	N/A
12	8	3	14	10.1999	30.4641	0.0102	N/A	N/A
12	8	3	15	11.5842	30.1839	0.0091	N/A	N/A

N/A: the solution couldn't be found.

As seen from Table 1, the maximum difference in objective function values between the decomposed IP and the exact methods is about 0.05%, which shows that the decomposed IP method was able to find a very good solution in significantly less amount of the time.

2.5.2 Simulation Experiment 2 (the LA/LB port area scenario)

In this simulation experiment, we consider the LA/LB port area. The geographical boundaries of the area are taken from (Jula *et al.*, 2003). Similar to (Jula *et al.*, 2003), we assume that there are 12 consignees (supply of empties), 8 shippers (demand for empties), 2 local container depots, and 1 container terminal located in the area. The degree of activity of each entity and the traveling distance between entities are based on the data used in (The Tioga Group, 2002, and Jula *et al.*, 2003).

In this scenario, we use the projected figures for the empties demand and supply for 2010 derived in (Jula *et al.*, 2006). It is expected that in 2010 the number of export and import loads in the LA/LB port area will be about 2.0 and 1.8 times more than those in the year 2000, respectively (The Tioga Group, 2002). Thus, the number of empties demanded by shippers will be around 534, and the number of empties supplied by consignees will be about 985 containers per day in 2010.

We assume that the above total daily numbers of empties are distributed among the consignees and shippers according to their degree of activity provided by (Jula *et al.*, 2006). We also assume that there are only three classes of containers t_1, t_2, and t_3, which are given by (2), in the LA/LB port area.

The number of empties from each class of containers demanded by each shipper is then assumed to be divided into two parts: the exact and substitutable requests. The exact requests are the requests for the exact types of containers which cannot be substituted by any other types. The substitutable requests are those which can be fulfilled by other empties and are chosen, with equal probability, from one of the following substitution rules:

1. Requests should be satisfied by exactly one specified types.

2. Requests could be satisfied by any type with the same dimension.

3. Requests could be satisfied by any type.

To evaluate the benefit of adapting the substitution mechanism, the cost c_{ij}^t is simply chosen to be the traveling distance between consignee $i \in I \bigcup P$ and shipper $j \in J \bigcup P$.

Table 2 shows the results when the ratio of the number of substitutable requests to the total number of requests at shippers is varied from 0% to 100%. When this rate is selected to be 0% the problem is, in fact, reduced to three single commodity problems (**P1s**), each for one type of containers. Since the exact BNB method is very slow in finding the solutions and experiences out-of-memory frequently, the method is not considered here.

Table 2. Cost reduction w.r.t. the substitutable requests ratio

Substitutable requests Ratio [%]	Combination of Types [%]						Relaxed LP		Decomposed IP	
	Supply			Demand						
	t1	t2	t3	t1	t2	t3	Cost [miles]	Cost Reduction [%]	Cost [miles]	Cost Reduction [%]
0	50	25	25	50	25	25	10235.83	0	D/N	0
20							9649.13	5.73	9657.77	5.65
40							9609.13	6.12	9619.08	6.03
60							9606.05	6.15	9617.18	6.04
80							9597.78	6.23	9615.61	6.06
100							9597.55	6.24	9611.91	6.1
0	50	25	25	25	25	50	10962.64	0	D/N	0
20							9801.63	10.59	9814.04	10.48
40							9629.49	12.16	9647.43	12
60							9607.52	12.36	9626.63	12.19
80							9597.55	12.45	9616.03	12.28
100							9597.55	12.45	9611.91	12.32
0	70	20	10	70	20	10	10235.83	0	D/N	0
20							9753.30	4.71	9757.35	4.67
40							9728.27	4.96	9735.49	4.89
60							9728.05	4.96	9733.01	4.91
80							9727.58	4.97	9736.83	4.88
100							9727.58	4.97	9738.20	4.86
0	70	20	10	20	10	70	17288.96	0	D/N	0
20							14720.94	14.85	14723.97	14.84
40							12365.06	28.48	12366.44	28.47
60							11005.61	36.34	11006.99	36.34
80							9413.19	45.55	9423.02	45.5
100							9381.49	45.74	9389.44	45.69
Base Operation							25,578.11			

D/N: don't need to run the IP since the LP has the integer solution

As reported by (Jula *et al.*, 2003 and 2006), the empty container reuse scenario results in more than 50% reduction in the empty trips activity around the port and depots in the LA/LB port area, as compared to the base scenario which represent the current practices (i.e. no depot-direct and street-turn exchange of empties). The cost of the base scenario is shown in the last row of the table.

In Table 2, the total empty traveling costs decreases as the total substitutable requests increases at shippers. When all the requests are substitutable, an additional cost reduction (compared to Jula et al., 2003), ranges from 5% to 46% for different scenarios, is obtained. Table 2 indicates that the higher the imbalance between the empty types is, the more reduction in the empty substitution cost can be achieved. Therefore, the cost reduction turns out to be more significant when substitution is allowed. In other words, the cost of empty container reuse can be further reduced by allowing the substitution between empty containers in the region. This can be translated into further reduction in the traffic and congestion around the port and therefore, further reduction in noise and emissions.

3. Empty container interchange problem in stochastic environments

In this section, we investigate the single commodity empty interchange problem in a stochastic environment. We assume that the supply of empties is deterministic and that the probability distributions of the demands are known and consist of finite sets of scenarios.

3.1 Empty container interchange with stochastic demand

Let the triplet (Ω, F, P) be a probability space, where F is a collection of events, $\Omega \in F$ is an event (the set of all possible scenarios), and P is the probability measure. Let ω be an outcome (i.e. a scenario) of event Ω which is a random experiment. Given the cost vector $\mathbf{c}$, the modified problem **P1** with stochastic demands can be expressed in the matrix-vector form as

$$\text{Minimize:} \qquad \mathbf{c}^T\mathbf{x} \tag{13}$$

$$\text{Subject to} \qquad \mathbf{A}_U\mathbf{x} = \mathbf{s} \tag{14}$$

$$\mathbf{A}_L\mathbf{x} = \mathbf{d}(\omega) \tag{15}$$

$$\mathbf{x} \geq \mathbf{0} \tag{16}$$

where $\mathbf{s}$ is the supply vector, and $\mathbf{d}(\omega)$ is the stochastic demand vector of all scenarios ω.

It should be noted that for a given decision vector $\mathbf{x}$ and a realization ω, constraints (15) should be met. To compensate for any constraints violation, we will provide a recourse vector **y,** that after observing the realization of ω, will affect the choice of $\mathbf{x}$. In other words, since the decision vector $\mathbf{x}$ in (13) must be made before the realization of ω is known, a second-stage linear program is introduced, whose values are uncertain but will influence the choice of $\mathbf{x}$.

Since every scenario may involve a different set of constraints, a more reasonable objective is to choose the decision variables so that the expected cost of the following recourse formulation is minimized.

$$\begin{aligned} \min\ & \mathbf{c}^T\mathbf{x} + E_\omega[\mathbf{q}^T\mathbf{y}] \\ s.t.\ & \mathbf{A}_U\mathbf{x} \qquad\qquad = \mathbf{s} \\ & \mathbf{A}_L\mathbf{x} \quad + \mathbf{W}\mathbf{y} = \mathbf{d}(\omega) \qquad \omega \in \Omega \\ & \qquad \mathbf{x},\ \mathbf{y} \geq \mathbf{0} \end{aligned} \tag{17}$$

where E_ω stands for the mathematical expectation that is the weighted average over all ω and $\mathbf{W} = \left[\mathbf{I}_m \quad -\mathbf{I}_m \right]$ is the recourse matrix; $\mathbf{I}_m$ is an identity matrix, and m is the number of shippers.

In the above formulation, exogenous variable $\mathbf{y}$ is used as the second stage variable and the recourse cost vector $\mathbf{q}^T$ is introduced to penalize constraints violations.

3.2 Monte Carlo sampling and VSS

Using the deterministic equivalent, the two-stage stochastic program can be solved by linear programming techniques. However, even for a moderate number of possible scenarios, the deterministic equivalent could result in a huge linear programming problem. For example, in the case of the stochastic transportation problem **P1,** we have assumed demands in the form of finite sets of scenarios. Since **P1** includes a set of depots which can be considered as super sources, every realization scenario ($\omega \in \Omega$) will be feasible, and hence, the number of feasible scenarios will be

$$|\Omega| = \prod_{j=1}^{n} \left| d_j(\omega) \right| \tag{18}$$

where n is the number of shippers, $d_j(\omega) = \left\{ d_{j1}, \cdots, d_{js_j} \right\}$ is the finite set of demands (scenarios) at shipper j, and s_j is the cardinality of the set $d_j(\omega)$.

In other words, although Ω is a finite set, the number of realizable scenarios is too many to enumerate all. One way to overcome this problem is to use decomposition methods, such as Bender’s decomposition method (Bertsimas and Tsitsiklis, 1997). Unfortunately, although a decomposition method could reduce the number of variables substantially, the method still generates extremely large number of constraints. For this reason, in this paper, we use a sampling technique called Monte Carlo simulation method to estimate the expected value of the stochastic program.

Consider the stochastic transportation problem **P1** in (17). Let v^* be the optimal expected value of the problem, which can be expressed as the following two-stage stochastic program.

$$v^* = \min_{x \in S} \left\{ f(\mathbf{x}) \equiv E_\omega \left[g\left(\mathbf{x}, \mathbf{d}(\omega)\right) \right] \right\} \tag{19}$$

where $S \equiv \left\{ \mathbf{x} \middle| \mathbf{A}_U \mathbf{x} = \mathbf{s},\ \mathbf{x} \geq 0 \right\}$ and $g\left(\mathbf{x}, \mathbf{d}(\omega)\right) \equiv \mathbf{c}^T \mathbf{x} + \min_{\mathbf{y} \geq 0} \left\{ \mathbf{q}^T \mathbf{y} \middle| \mathbf{W}\mathbf{y} = \mathbf{d}(\omega) - \mathbf{A}_L \mathbf{x} \right\}$.

Let random samples $\mathbf{d}^1, \cdots, \mathbf{d}^N$ be N realizations of the random vector $\mathbf{d}(\omega)$, and $\hat{f}_N(\mathbf{x}) \equiv N^{-1} \sum_{k=1}^{N} g(\mathbf{x}, \mathbf{d}^k)$ be the sample average approximation (SAA) of $f(\mathbf{x})$. By replacing $f(\mathbf{x})$ with $\hat{f}_N(\mathbf{x})$ in (19), we find the optimal expected value of the *approximated* stochastic problem by

$$\hat{v}_N = \min_{x \in S} \left\{ \hat{f}_N(\mathbf{x}) \equiv N^{-1} \sum_{k=1}^{N} g(\mathbf{x}, \mathbf{d}^k) \right\} \tag{20}$$

Since the random realizations $\mathbf{d}^1, \cdots, \mathbf{d}^N$ have the same probability distribution as $\mathbf{d}(\omega)$, it follows that $\hat{f}_N(\mathbf{x})$ is an unbiased estimator of $f(\mathbf{x})$ for any $\mathbf{x}$ (Linderoth, 2002). By generating M independent sample sets $\mathbf{D}^j = \left\{ \mathbf{d}^{1,j}, \cdots, \mathbf{d}^{N,j} \right\}$, j=1,…,M, each of size N, and solving the corresponding SAA problems in (20), the sample average of the optimal expected values of the approximated stochastic programs can be computed by

$$L_{N.M} = M^{-1} \sum_{j=1}^{M} \hat{v}_N^j \tag{21}$$

where $\hat{v}_N^j$ is the optimal expected value of SAA problem in (20) for each sample set $\mathbf{D}^j$. It can be shown that $L_{N.M} = M^{-1}\sum_{j=1}^{M}\hat{v}_N^j$ is an unbiased estimator of $E\left[\hat{v}_N\right]$ (Lineroth, 2002).

In stochastic optimization problems, the value of the stochastic solution (VSS) is defined as the difference between the optimal expected value of the stochastic problem and the solution of the deterministic equivalent computed by replacing stochastic variables by their mathematical expectations. The former is called the stochastic solution (SS) which is the solution of (20) and the latter is called the expected value (EV) solution. VSS indicates the benefit of knowing the distributions of the stochastic variables (Birge and Louveaux, 1997).

In particular, EV can be computed by taking the following procedure. During the first stage, a super-model solution is computed using the expected demand. Subsequently, with the first stage values fixed, each sub-model solution is independently computed and averaged over all sampled scenarios.

Let $\overline{\mathbf{x}}$ be the solution to the supermodel which is constructed by replacing random variables by their expectations. Hence, the expected value (EV) is obtained by

$$v_E = \min_{x\in S}\left\{ f(\mathbf{x}) = g\left(\mathbf{x}, E_\omega\left[\mathbf{d}(\omega)\right]\right)\right\} = g\left(\overline{\mathbf{x}}, \mu\right) \tag{22}$$

where $\mu = E_\omega\left[\mathbf{d}(\omega)\right]$.

We generate N independent random samples $\mathbf{d}^1,\cdots,\mathbf{d}^N$ of $\mathbf{d}(\omega)$. For each $\mathbf{d}^k$, k=1,…,N, we compute

$$v^k = g\left(\overline{\mathbf{x}}, \mathbf{d}^k\right) \tag{23}$$

where $g\left(\overline{\mathbf{x}},\mathbf{d}^k\right) \equiv \mathbf{c}^T\overline{\mathbf{x}} + \min_{\mathbf{y}\geq 0}\left\{\mathbf{q}^T\mathbf{y} \,\middle|\, \mathbf{W}\mathbf{y} = \mathbf{d}^k - \mathbf{A}_L\overline{\mathbf{x}}\right\}$. The expectation of the EV (EEV) can be estimated by obtaining the sample average of $v^k = g\left(\overline{\mathbf{x}},\mathbf{d}^k\right)$ over all the sampled scenarios, i.e.,

$$E\left[\hat{v}_E\right] = N^{-1}\sum_{k=1}^{N} g\left(\overline{\mathbf{x}},\mathbf{d}^k\right) \tag{24}$$

Usually, if the difference

$$EEV - EV = E\left[\hat{v}_E\right] - v_E \tag{25}$$

is small, EEV is said to be a reasonably good solution to the stochastic program (Birge and Louveanux, 1997). Furthermore, the value of the approximated stochastic solution (VSS) is computed by

$$VSS = EEV - ESS = E\left[\hat{v}_E\right] - E\left[\hat{v}_N\right] \tag{26}$$

This value of VSS in (26) indicates the price of using naive EV model instead of SS.

3.3 Simulation experiments

In this section, we perform a series of simulation experiments to evaluate the stochastic solution (SS) and the expected value solution (EV) of the stochastic empty container interchange problem. We assume that the distributions of empty demands are known and that they consist of finite sets of scenarios. These stochastic demands include the last-minute empty requests and cancellations within the working day.

Consider the simulation scenario in Section 3 used for the LA/LB port area. Again, we use the projected figures for the empties demand and supply for 2010. Let d_j be the number of empties needed at shipper *j* obtained by applying the aforementioned procedure. We assume that the actual number of empty requested at shipper *j* is a discrete random number given by

$$d_j(\omega) = d_j + \omega \tag{27}$$

where ω is assumed to have the discrete uniform distribution, which can take any number with equal probability from the integer set

$$I_n = \{0, 1, \ldots, n\}. \tag{28}$$

Table 3 shows the optimal value for the expected value solution (EV) for different demand sets generated from different I_n. The Monte Carlo sampling method discussed in (23) and (24) is used to determine the expectation of the EV (EEV) when *N* is varied from 100 to 10,000. The third column presents the worst-case solution which is sometimes referred to as a *fat solution*. The fat solution will be the feasible solution for all possible realizations of ω in (27). More precisely, we let $\omega = n$, where n is the maximum possible realization of ω and is given in (28). The worst-case scenario is, then, solved as a single-commodity deterministic transportation problem.

Similar to the second experiment in Section 3, the cost of moving an empty between a consignee and a shipper is assumed to be the traveling distance between these two nodes, thus has a deterministic value. As a comparison purpose, the cost of the base operation scenario is included in the last row of each set when $N = 10000$.

Table 3. Expected value solution of the approximating stochastic program

Demand Set	N	worst-case [miles]	Expected value solution		$\frac{\text{EEV - EV}}{\text{EV}}$ [%]
			EV [miles]	EEV [miles]	
Set 1	100	10614.81	10291.35	10514.43	2.1676
	200			10516.71	2.1898

$I_5 = \{0, \cdots, 5\}$	300			10518.57	2.2079
	1000			10519.66	2.2185
	10000			10515.41	2.1772
Base operation	10000	25909.29	25515.04	25516.10	0.0042
Set 2 $I_{10} = \{0, \cdots, 10\}$	100	10633.98	10240.45	10505.59	2.5891
	200			10505.38	2.5871
	300			10505.59	2.5891
	1000			10503.82	2.5719
	10000			10504.32	2.5767
Base operation	10000	26053.93	25574.34	25574.76	0.0016
Set 3 $I_{15} = \{0, \cdots, 15\}$	100	10727.65	10018.77	10460.43	4.4083
	200			10471.15	4.5153
	300			10469.01	4.4940
	1000			10466.62	4.4701
	10000			10469.62	4.5001
Base operation	10000	26669.04	25822.99	25823.61	0.0024

Table 3 indicates that the worst-case solution is very expensive, and that the difference between EEV and EV is fairly small about 2 to 5%. Usually, if the difference is small, the EEV is reasonably good approximation to the solution of the stochastic problem (Birge and Louveaux, 1997). It also indicates that the difference between EEV and EV is not sensitive to the sampling size N . Therefore, even the smallest independent sample size ($N = 100$) can be considered sufficient to capture the stochastic behavior of the demand in our stochastic transportation problem.

In Table 4, we calculate the estimated stochastic solution (ESS) using the Monte Carlo sampling method for different demand sets generated from different I_n .The ESS is compared with the EEV when the sampling size N is 100, 200, and 300.

Table 4. Stochastic solution of the approximation stochastic program

Demand Set	N	EEV [miles]	confidence interval [miles]		ESS [miles] (M=10)	VSS [miles]	$\frac{VSS}{ESS}$ [%]
			90%	95%			
Set 1 $I_5 = \{0, \cdots, 5\}$	100	10514.43	±16.45	±19.66	10503.62	10.81	0.1029
	200	10516.71	±11.84	±14.14	10508.21	8.50	0.0809
	300	10518.57	±9.62	±11.47	10512.62	5.95	0.0566
Set 2 $I_{10} = \{0, \cdots, 10\}$	100	10505.59	±20.35	±24.32	10489.42	16.27	0.1551
	200	10505.38	±14.46	±17.26	10499.50	5.88	0.0560
	300	10505.59	±10.92	±13.03	10497.46	8.13	0.0774
Set 3 $I_{15} = \{0, \cdots, 15\}$	100	10460.43	±34.56	±41.30	10453.63	6.80	0.0650
	200	10471.15	±24.35	±29.06	10454.39	16.76	0.1603
	300	10469.01	±21.38	±25.50	10452.77	16.24	0.1554

The iteration number M for samplings is chosen to be 10 so that the ESS does not pass the 90% confidence interval. Table 4 shows the value of the approximated stochastic solution (VSS) given by (26). The VSS is very small relative to the total travel miles (about 0.2%). Therefore, one can approximate, with relatively small error, the expected value solution (EEV) as the optimal solution that minimizes the expected cost of the stochastic program.

5. CONCLUSIONS

We investigated the multi-commodity empty container substitution problem, in which one type of containers can be substituted with another container. We modeled the problem analytically and proposed an optimization technique to find sub-optimal solutions. In order to investigate the efficiency of the proposed optimization techniques we developed a realistic simulation scenario using past, current, and projected data in the Los Angeles/Long Beach (LA/LB) port area. Compared to single commodity empty container reuse problem, the simulation results demonstrate that the cost of empty container reuse can be further reduced by allowing the substitution between empty containers in the LA/LB port area. In our simulation experiments, we observed an additional cost reduction in the range of 5% to 46%. The amount of reduction was mainly dependent on the combination of the container types in the supply and demand nodes.

Furthermore, in this paper, the effect of uncertainties associated with demand requirements in the stochastic empty container interchange problem was investigated. We modeled the problem as a stochastic problem with recourse in order to minimize the expected cost of interchanging empty containers. The results showed that our approach can also perform well in a stochastic environment.

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