Pickup and delivery problem with hard time windows considering stochastic and time-dependent travel times

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Motivation

Introduction to pickup and delivery problems (PDP):

- A generalization of vehicle routing problems (VRP)
- With pickup and delivery pairs visited by the same vehicle
- Applications: ridesharing, food delivery, grocery delivery

Motivation:

- The travel time in urban area is unpredictable, and fluctuates with the time of day
- How to guarantee the reliability of pickup and delivery services?

Literature Review

- Vehicle routing problems considering uncertainty:
  Stochastic vehicle routing problems
  Robust vehicle routing problems

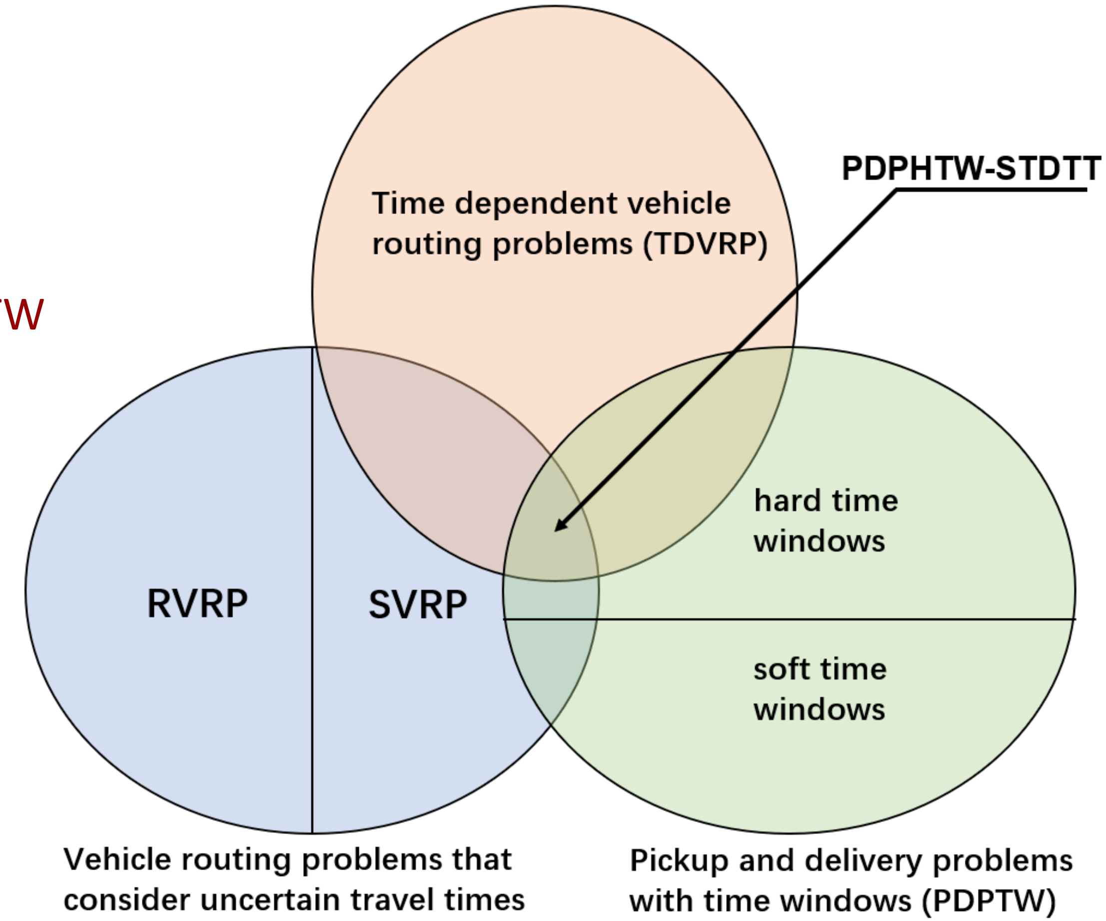
- Time windows
  Soft time windows
  Hard time windows

- Method to model time dependent travel time
  First-In-First-Out(FIFO) assumption

Literature Review

- The position of the proposed problem in the literature

RVRP=Robust VRP
SVRP= Stochastic VRP
PDPHTW=Pick up and Delivery Problem with HTW
STDTT=Stoch. Time Dependent Travel Times



Problem Definition

- A capacitated pickup and delivery problem:
  - $2n + 1$ Nodes. The depot is node 0.
  - $1 \sim n$ pickup nodes. $n + 1 \sim 2n$ corresponding delivery nodes.
  - Each node i associated with a load q_i .
  - Each node i associated with a time window $[e_i, l_i]$.

- Stochastic and time dependent travel times
  - The travel time on each arc (i, j) is a Gaussian random process $X_{ij}(t)$.
  - Assume the mean of $X_{ij}(t)$, $E[X_{ij}(t)]$ is time-dependent.
  - $E[X_{ij}(t)]$ is a piecewise linear function.
  - Assume $Var[X_{ij}(t)]$ is proportional to the travel distance.

Problem Definition

- A route r is defined by a sequence of nodes, which starts from the depot and ends at the depot.

- A node is successful if the vehicle arrives within its time window.
  A route is successful if all the nodes on the route are successful.

- Node service level: desired successful probability of a node
  Global service level: desired probability of all the nodes is successful

Notations

θ_i: the desired service level of node i.

Θ: the desired global service level.

A_i^r: the arrival time at node i on route r.

D_i^r: the departure time at node i on route r.

S_i: the service time of node i. (stochastic, time independent)

P_i^r: the node success probability at node i on route r.

P^r: the route success probability of route r.

c_r: the cost of route r.

$X_{ij}(t)$: the travel time on arc (i, j).

N: the set of all the nodes.

N_p: the set of pickup nodes.

R: the set of all feasible routes.

* A route is considered feasible when all the node service levels of the nodes on the route, and all constraints in the pickup and delivery problem are satisfied.

Methodology

1. Route success probability estimation

Jula et al.[1] proposed an arrival time estimation method. Given a route, the mean and variance of node arrival times and departure times can be estimated iteratively.

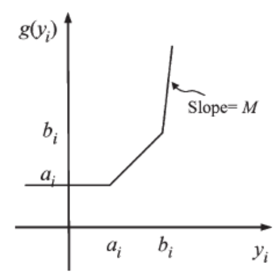
$$E[A_j^r] \approx E[D_i^r] + E[X_{ij}(E[D_i^r])]$$

$$Var[A_j^r] \approx \{1 + E'[X_{ij}(E[D_i^r])]\}^2 Var[D_i^r] + Var[X_{ij}]$$

$$E(D_j^r) = g_j(E[A_j^r]) + E[S_j]$$

$$Var(D_j^r) \approx \frac{1}{4}\left(\int_{E[A_j^r]-\sigma[A_j^r]}^{E[A_j^r]+\sigma[A_j^r]} g_j'(x)\,dx\right)^2 + Var[S_j]$$

where $g_i(\cdot)$ is a piecewise linear function:



[1]Jula, Hossein, Maged Dessouky, and Petros A. Ioannou. "Truck route planning in nonstationary stochastic networks with time windows at customer locations." *IEEE Transactions on intelligent transportation systems* 7.1 (2006): 51-62.

Methodology

1. Route success probability estimation

Given the node arrival time and departure time, the node success probability is estimated by:

$$P_i^r = P(A_i^r \le l_i) \approx \Phi\left(\frac{l_i - E[A_i^r]}{\sqrt{Var[A_i^r]}}\right)$$

Given the nodes on a route, the route success probability is given by:

$$P^r = \prod_{i \in r} P_i^r$$

Methodology

2. Problem formulation

The set partitioning formulation is given by:

$$minimize \quad \sum_{r \in \mathcal{R}} c_r x_r$$

$$s.t \quad \sum_{r \in \mathcal{R}} a_{ir} x_r = 1 \qquad \forall i \in N_p \qquad \text{---------- (1)}$$

$$\sum_{r \in \mathcal{R}} x_r \, ln(P^r) \geq ln(\Theta) \qquad \text{---------- (2)}$$

$$x_r \in \{0, 1\} \qquad \forall r \in \mathcal{R}$$

where a_{ir} is a binary variable indicating if node i is visited on route r.

The optimization objective is to minimize total operational cost.
Constraint (1) states that each pickup node is visited exactly once.
Constraint (2) ensures the global service level is satisfied.

Methodology

3. Branch-and-price solution framework

- The set of all feasible routes, R, is large and cannot be explicitly enumerated. Only a subset of R is maintained in the linear program. Column generation method is utilized to generate promising routes iteratively, and add these routes to the subset.

- A pricing algorithm is called iteratively to find routes with negative reduced cost. In the pricing algorithm, feasible routes are generated via dynamic programming. Only the most promising routes are added to the subset above.

- An important contribution is proposing route elimination rules for comparing which route is more promising with the presence of probabilistic information in the route. Elimination unpromising routes is important for accelerating the algorithm.

Methodology
4. Column generation

The reduced cost $\bar{c_r}$ of a route $r \in R$ is calculated as:

$$\bar{c}_r = c_r - \sum_{i \in N_p} a_{ir}\pi_i + ln(P^r)\lambda$$

where π_i and λ are the dual multipliers of constraints (1) and (2) in the linear program.

*The details of the pricing algorithm can be found in our paper.

Computational Experiments

- Experiment are conducted based on Ropke[2] dataset, a standard dataset for PDP. The number of nodes ranges from 40 to 90.

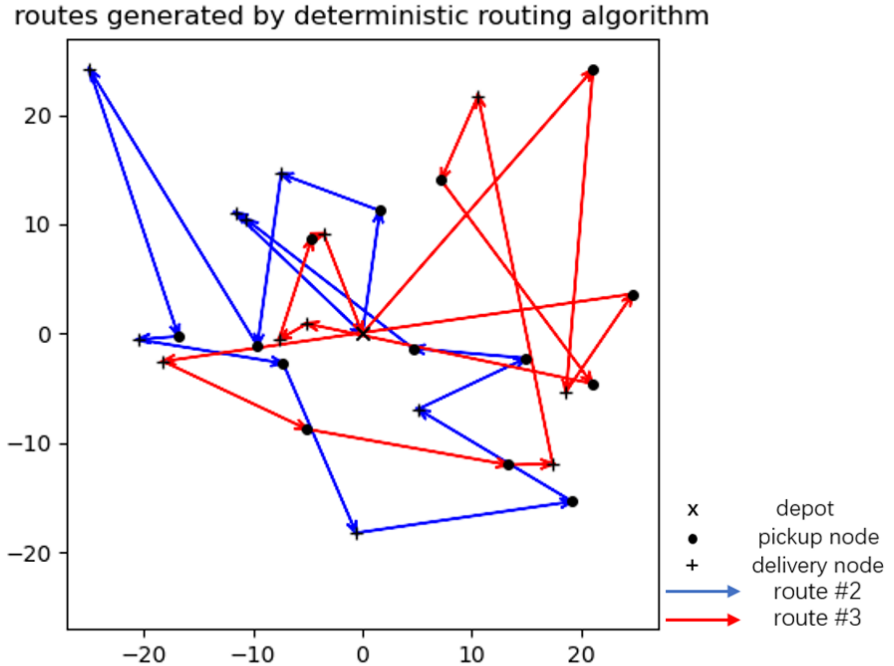
- Monte Carlo simulations are conducted to determine the actual success probabilities. The average estimation error between the estimated probability and the simulated probability is less than 10%.

- Compared to a deterministic algorithm that does not consider stochastic travel times, the proposed algorithm is able to guarantee the desired service levels, but at the cost of a higher operational cost.

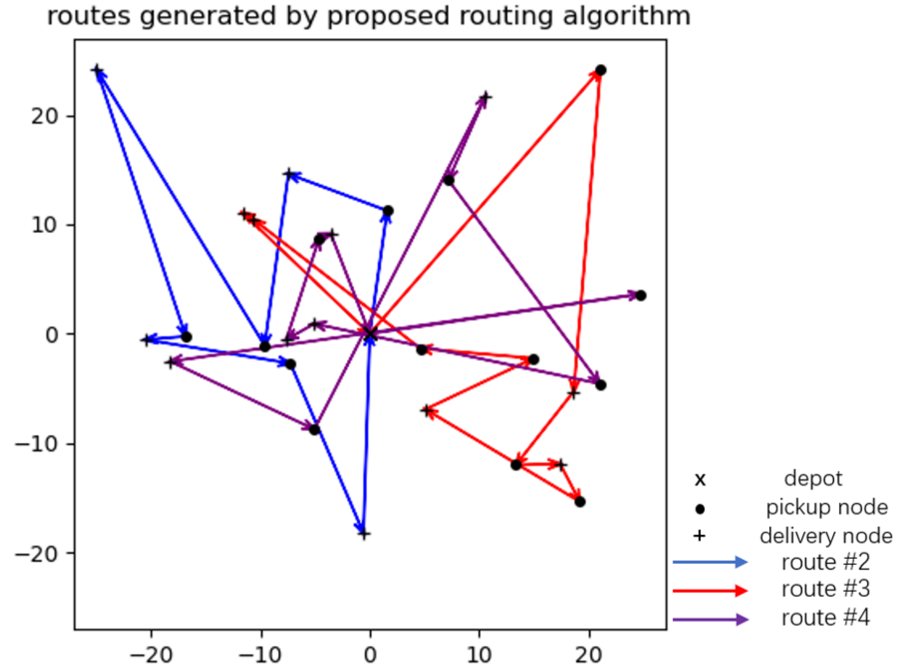
[2] Ropke, Stefan, Jean-François Cordeau, and Gilbert Laporte. "Models and branch-and-cut algorithms for pickup and delivery problems with time windows." *Networks: An International Journal* 49.4 (2007): 258-272.

Computational Experiments

An example that shows how the proposed algorithm enhances route reliability.
In the figure, each dot represents the location of a node.



Total cost 30751.71
Simulated global success probability 27.4%



Total cost 40757.84
Simulated global success probability 79.2%

Computational Experiments

number of total nodes	c_v	time window width	desired global service level	number of vehicles	cost	estimated global success probability	simulated global success probability	time(ms)
50	0.3	80	0.6	4	40980.69	69.07%	79.6%	22012
			0.7	4	40984.27	70.47%	78.0%	21440
			0.8	4	41011.97	80.43%	80.6%	48496
			0.9	5	50997.64	93.15%	99.5%	32665
60			0.6	5	51124.92	63.99%	78.3%	46997
			0.7	5	51124.51	70.36%	79.6%	38488
			0.8	5	51158.50	80.47%	83.2%	117882
			0.9	5	51174.69	90.12%	98.7%	79416
70			0.6	5	51298.32	63.34%	78.2%	919768
			0.7	5	51299.06	74.28%	91.2%	474662
			0.8	5	51323.53	85.60%	90.7%	1167582
			0.9	6	61325.10	90.03%	95.2%	167783
80			0.6	6	61484.20	62.86%	68.3%	550447
			0.7	6	61490.87	73.57%	89.7%	132978
			0.8	6	61499.46	84.41%	95.1%	357319
			0.9	6	61546.67	90.73%	94.8%	323820
90			0.6	6	61689.53	62.18%	79.9%	597232
			0.7	6	61691.90	74.24%	80.7%	1355956
			0.8	6	61705.41	81.21%	87.3%	1032443
			0.9	10	101698.51	90.06%	94.7%	1509237

Computational Experiments

Discussions:

- Experiments are conducted on instances with up to 90 nodes.
- There is a gap between the estimated global success probability and the simulated global success probability. The gap exists because there is no closed form of the real probability distribution of arrival times, and the estimation method has estimation error. The average gap in all instances is about 9.9%.
- In the vast majority of the cases, the estimation method gives a lower bound of the estimated probability. Therefore, the existence of the gap makes the simulated success probability even higher than the desired service level, making the proposed algorithm more reliable and more robust to fluctuations.

Computational Experiments

More insights:

- Routing algorithms that do not consider stochastic travel times perform poorly in terms of nodes and global success probability. However, there is a trade-off in operational cost and delivery reliability in terms of success probability.

- To achieve reliable and punctual service with a lower cost, it is practical to give the customers wider time windows if possible.

- Being able to model the change in travel time during the day is helpful in developing routes for improving the reliability of the delivery. Not considering time dependent travel time may lead to a lower cost, but it also results in a much lower success probability.

Contributions

- We define the pickup and delivery problem with hard time windows considering stochastic and time-dependent travel times.
- We present an algorithm which is based on branch-cut-and-price to solve the proposed problem. In the pricing problem, new labeling algorithm and dominance rules are proposed to deal with stochastic travel times and probabilistic information.
- In the numerical experiments, we evaluate the trade-off between the reliability and the total cost of the routing solution. We also demonstrate the algorithm’s effectiveness of guaranteeing desired service levels.

CONCLUSIONS

- **PDP is very important due to the trend in home deliveries ride sharing etc**
- **Ability to meet estimated delivery times are very crucial**
- **Hard Time Windows need to be met and is a measure of**
- **quality of service**
- **To meet the above challenges the stochasticity of travel**
- **times are very important**
- **Deterministic travel times in estimating times of arrivals**
- **and service may cut cost but affect quality of service**
- **Meeting hard time windows with a high quality of service may be more costly but it is possible by taking into account travel uncertainties**