Congestion Reduction via Personalized Incentives

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Motivation and Background

❖ Traffic congestion cost in US in 2019: **$88 billion**[1]

❖ Longer traffic can worsen the air quality [2]

❖ Strategies to solve traffic congestion[3]

1. Adding more capacity
2. Transportation System Management and Operation (TSM)
3. **Demand management**

❖ Road pricing policy

- Pros: in theory and some cases work
- Cons: equity barriers

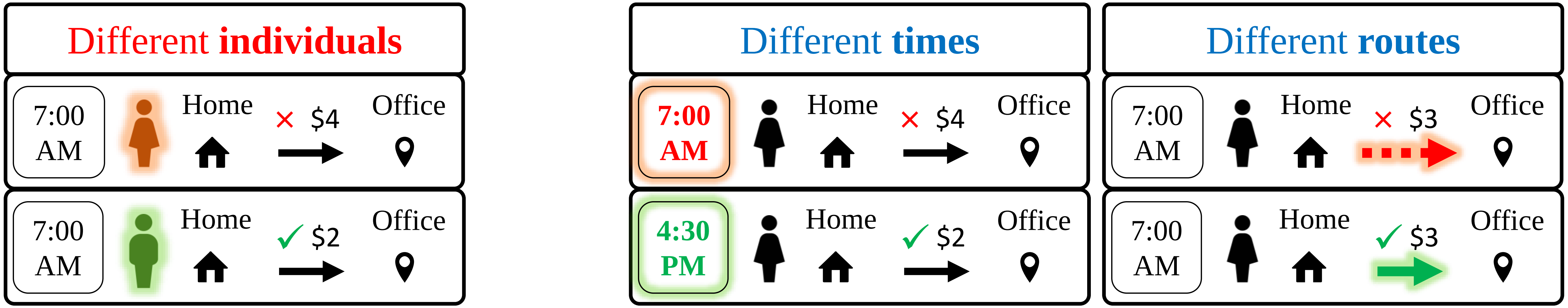
❖ Rewarding policy (positive incentive)

- ✓ Three projects in the Netherlands[4]
- ✓ CAPRI project[5]
- ✓ This research project

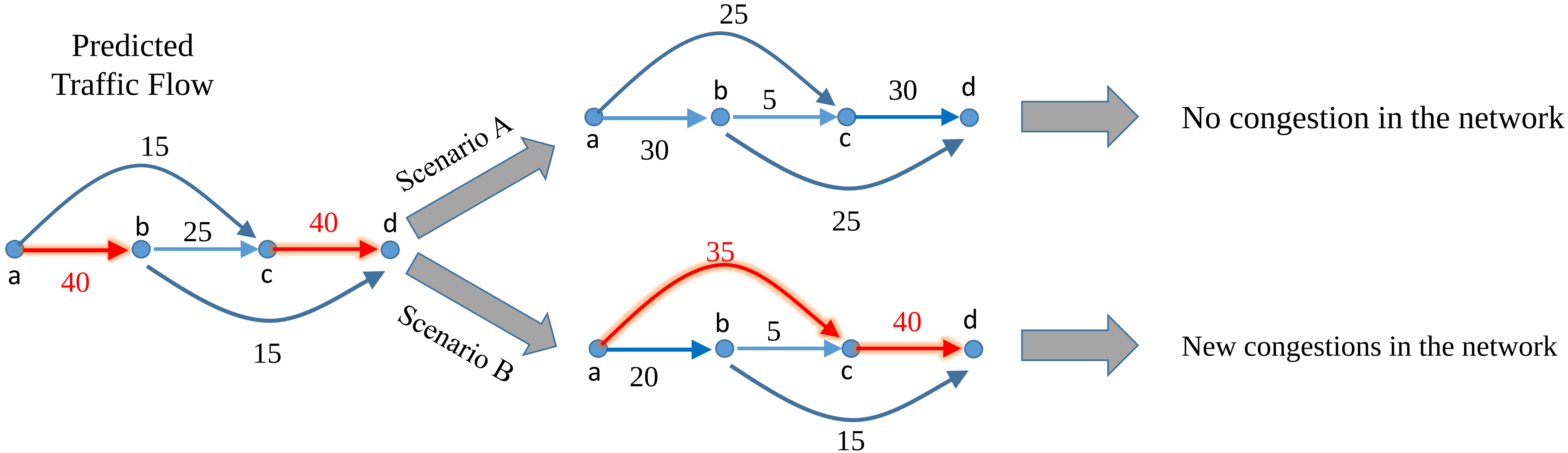
[1] Inrix 2018 global traffic scorecard. https://inrix.com/scorecard
[2] Health Effects Institute. Panel on the Health Effects of Traffic-Related Air Pollution. Traffic-related air pollution: a critical review of the literature on emissions, exposure, and health effects . Number 17. Health Effects Institute, 2010.
[3] Cambridge Systematics. Traffic congestion and reliability: Trends and advanced strategies for congestion mitigation. Technical report, United States. Federal Highway Administration, 2005.
[4] Michiel Bliemer, et. al., Rewarding for avoiding the peak period: A synthesis of three studies in the Netherlands. 2009.
[5] Jia Shuo Yue, Chinmoy V Mandayam, Deepak Merugu, Hossein Karkeh Abadi, and Balaji Prabhakar. Reducing road congestion through incentives: a case study. 2015.

Incentive Offering Process

❖ Personalized and Dynamic

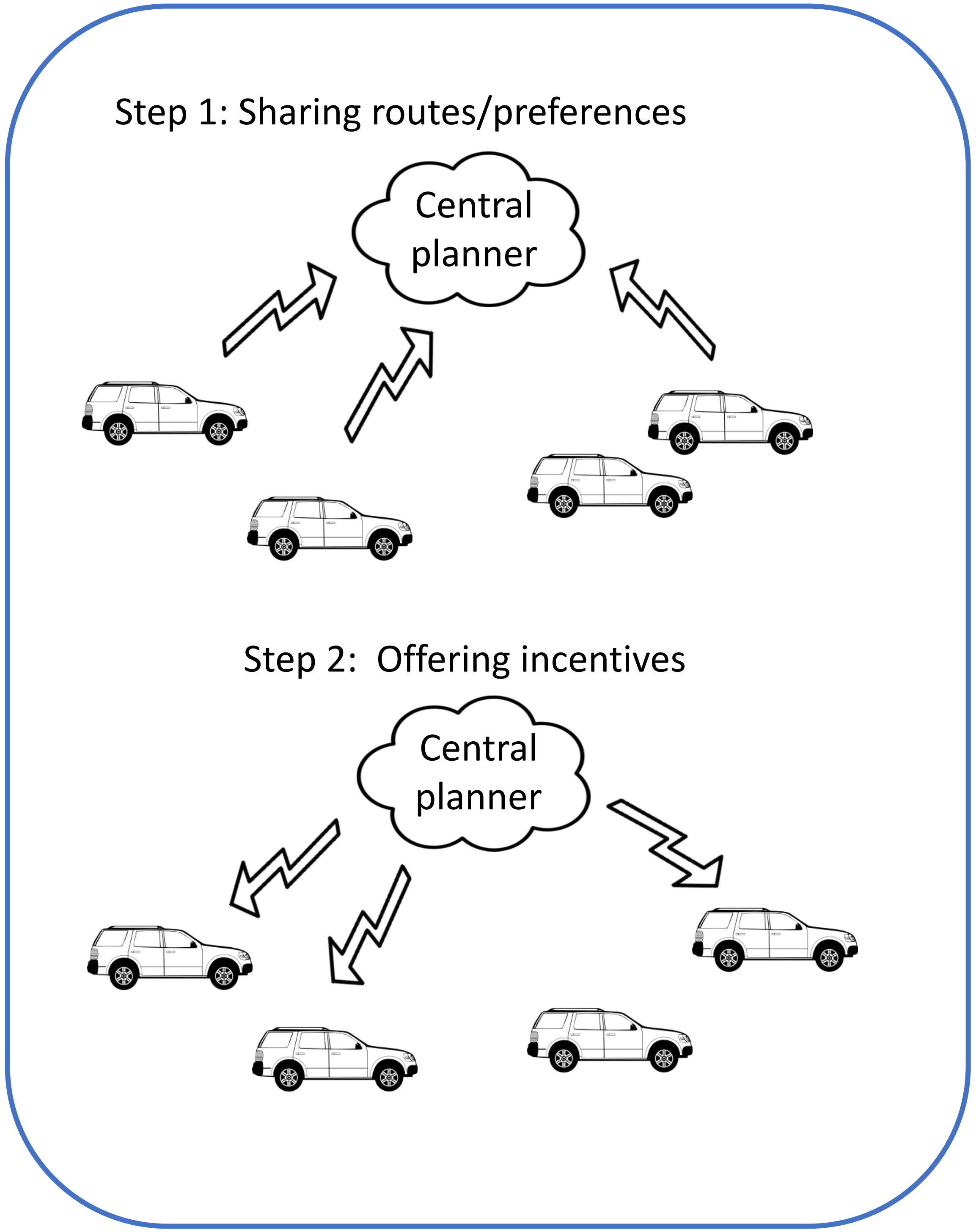


❖ Avoid creating new congestion

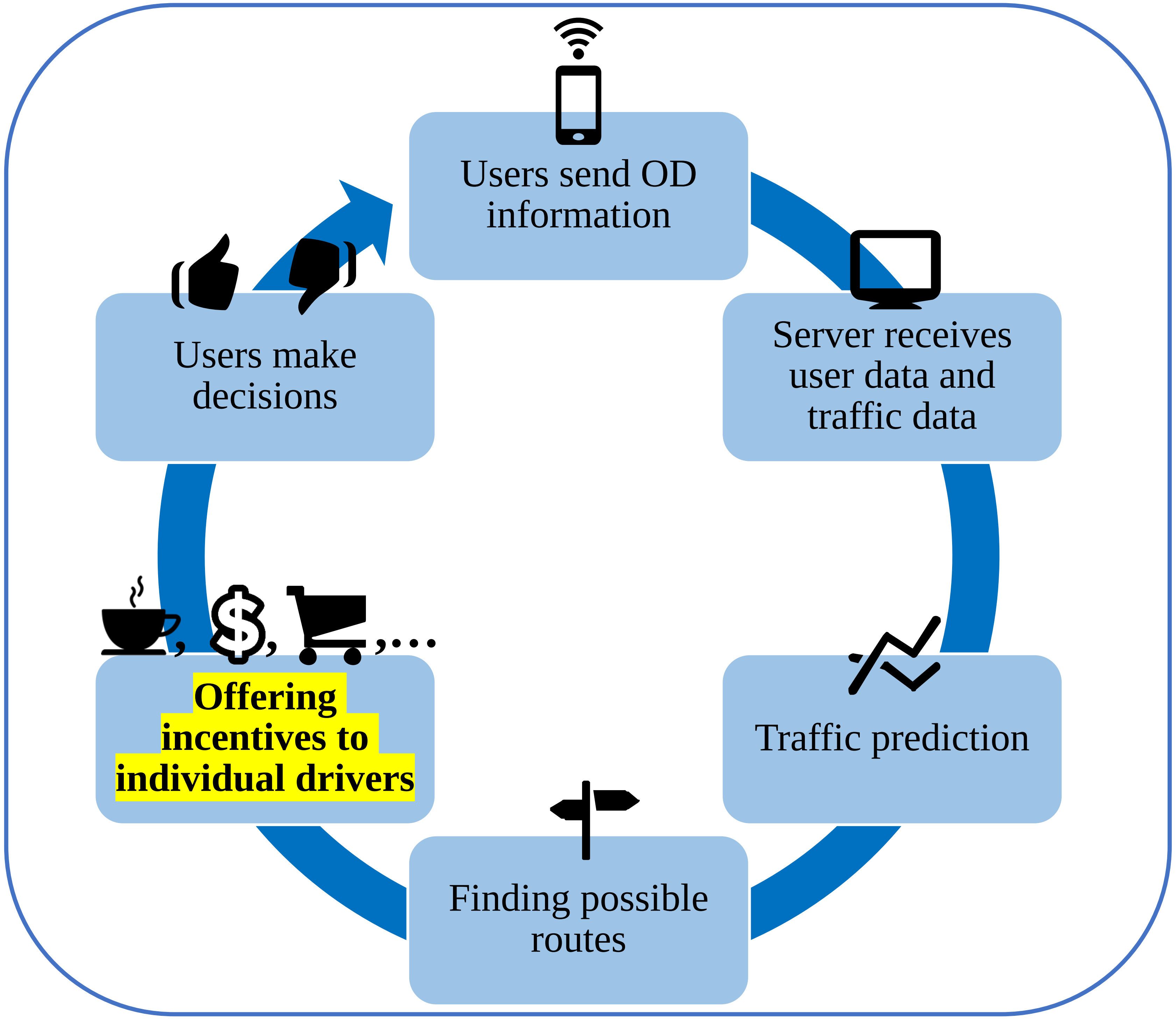


Incentivizing Process

High level process



Detailed process



Modeling

What should be our objective/goal?

- Minimize incentivizing cost
- Maximize a utility of the drivers' travel times
- Minimize Carbon emission footprint

A simple formulation

$$\begin{aligned} \min_{\text{incentives}} \quad & \text{Cost of offering incentives} \\ \text{s.t.} \quad & \text{Volume}_t \leq \text{Capacity}, \quad \forall t \end{aligned}$$

↓ Drivers' responses are random variables

$$\begin{aligned} \min_{\text{incentives}} \quad & \text{Cost of offering incentives} \\ \text{s.t.} \quad & \mathbb{E}\left[\text{Volume}_t\right] \leq \text{Capacity}, \quad \forall t \end{aligned}$$

First Model

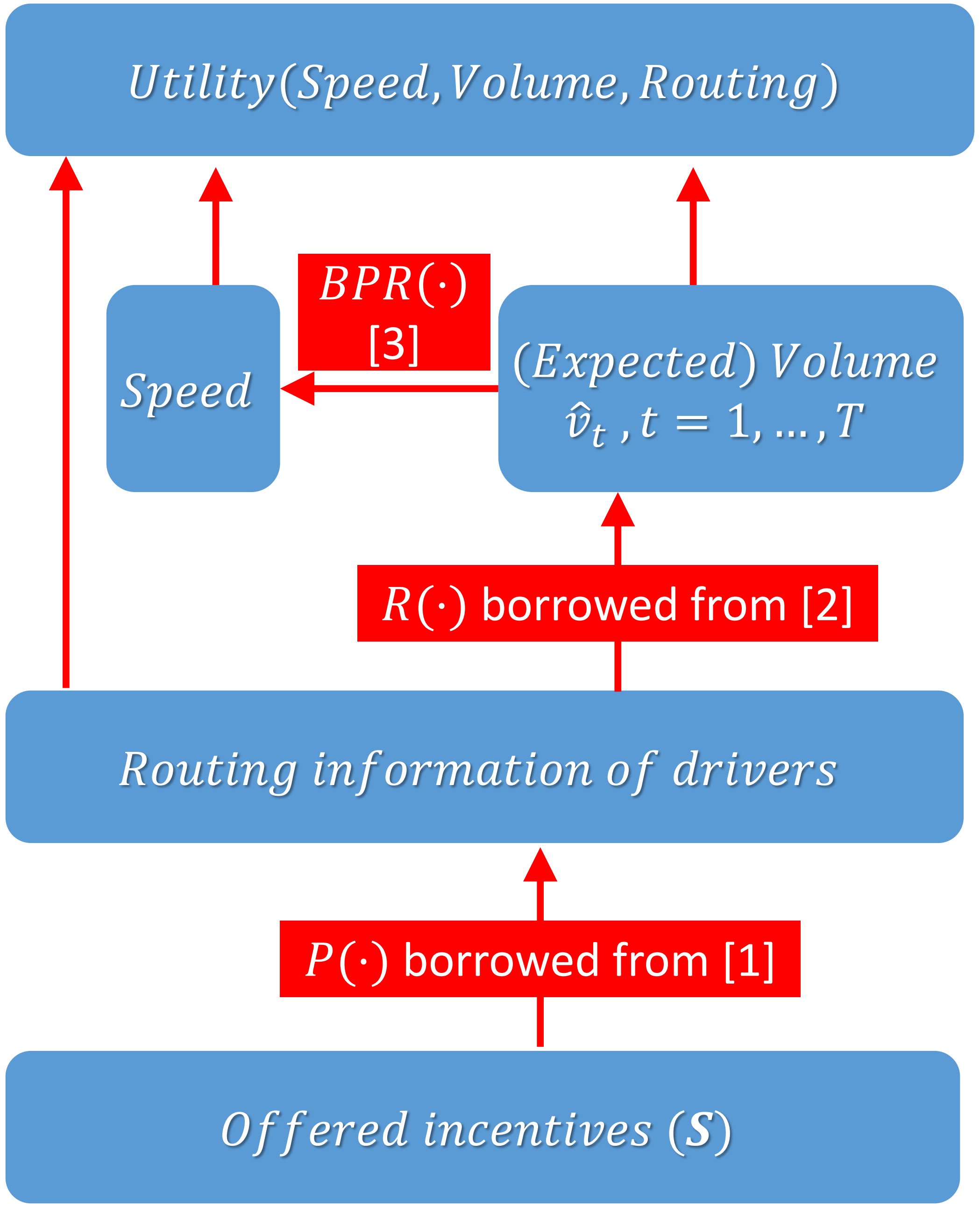
$$\min_{\text{incentives}} \quad \text{Cost of offering incentives}$$
$$\text{s.t.} \quad \mathbb{E}\left[\text{Volume}_t\right] \leq \text{Capacity}, \ \forall t$$
$$\text{constraints on incentive offering mechanism}$$

- Pros: ILP $\rightarrow$ off-the-shelf solvers
- Cons:
  - Is it fair?
  - It assumes feasibility.

$$\max_{\text{incentives}} \quad U(\text{Drivers' travel time})$$
$$\text{s.t.} \quad \mathbb{E}[\text{Volume}_t] \leq \text{Capacity}, \quad \forall t$$
$$\text{Cost of offering incentives} \leq \text{Budget}$$
$$\text{Other constraints incentives}$$

- Major (limiting) assumption:
  - We are operating below the system capacity (feasibility).

Operating in Congested Networks



Example: Use the total carbon emission as the objective

$$\min_{\mathbf{S}} \sum_{\ell=1}^{|\mathcal{E}|} \sum_{t=1}^{|\mathbf{T}|} \hat{v}_{\ell,t} f_{CE}\left(\hat{v}_{\ell,t}\right) L_\ell$$ → Total Carbon Emission [3,4]

$$\text{s.t.} \quad \hat{v}_{\ell,t} = (\mathbf{RP})_{\ell,t}\mathbf{S1}$$ → Estimated volume [1,2]

$$\mathbf{S}^\intercal \mathbf{1} = \mathbf{1}$$ → One incentive per driver

$$\mathbf{c}^\intercal \mathbf{S1} \leq \Omega$$ → Budget constraint

$$\mathbf{DS1} = \mathbf{q}$$ → Aware of the # of drivers per O-D

$$\mathbf{S} \in \{0,1\}^{(|\mathcal{R}||\mathcal{I}|)\times|\mathcal{N}|}$$

- Modular Design
  - Can be changed if needed
  - Can be learned
    - Use preference learning
    - Parameterize by a neural network and learn
- **How to solve it? Large-scale and challenging**

[1] Chenfeng Xiong, Mehrdad Shahabi, Jun Zhao, Yafeng Yin, Xuesong Zhou, and Lei Zhang. An integrated and personalized traveler information and incentive scheme for energy efficient mobility systems. Transportation Research Part C: Emerging Technologies, 2019.

[2] Wei Ma and Zhen Sean Qian. Estimating multi-year 24/7 origin-destination demand using high-granular multi-source traffic data. Transportation Research Part C: Emerging Technologies, 96:96–121, 2018.

[3] United States. Bureau of Public Roads. Traffic assignment manual for application with a large, highspeed computer, volume 37. US Department of Commerce, Bureau of Public Roads, Office of Planning, Urban Planning Division, 1964.

[4] PG Boulter and IS McCrae. Artemis: Assessment and reliability of transport emission models and inventory systems-final report. TRL Published Project Report, 2007.

Efficient Algorithm

National Center for Sustainable Transportation

$$\min_{\mathbf{S}} \sum_{\ell=1}^{|\mathcal{E}|} \sum_{t=1}^{|\mathbf{T}|} \hat{v}_{\ell,t} f_{CE}\left(\delta(\hat{v}_{\ell,t})\right) L_\ell$$ → Total Carbon Emission [3,4]

$$\text{s.t.} \quad \hat{v}_{\ell,t} = (\mathbf{RP})_{\ell,t}\mathbf{S1}$$ → Estimated volume [1,2]

$$\mathbf{S}^T\mathbf{1} = \mathbf{1}$$ → One incentive per driver

$$\mathbf{c}^T\mathbf{S1} \leq \Omega$$ → Budget constraint

$$\mathbf{DS1} = \mathbf{q}$$ → Aware of the # of drivers per O-D

$$\mathbf{S} \in \{0,1\}^{(|\mathcal{R}||\mathcal{I}|)\times|\mathcal{N}|}$$

$$f_{CE}(\delta) = 523.7 - (1654.4 \times 10^{-2})\delta - (2635.4 \times 10^{-4})\delta^2 - (1771.5 \times 10^{-6})\delta^3 - (442.9 \times 10^{-8})\delta^4,$$ [4]

$$\delta(v) = \frac{L}{t_0}\left(1 + 0.15\left(\frac{v}{w}\right)^4\right)^{-1}$$ [3]

Theorem: Relaxing the last constraint leads to a convex optimization problem!

- How should we solve this problem?
  - First order methods
  - Off-the-shelf solvers such as CVX and Gurobi
- It is still challenging due to massive scale of the problem.
- Can we use distributed/edge computation?
- Can we exploit the individual processing power of drivers' smartphones?
- We use Alternating Direction Method of Multipliers (ADMM) to do distributed computation.

[1] Chenfeng Xiong, Mehrdad Shahabi, Jun Zhao, Yafeng Yin, Xuesong Zhou, and Lei Zhang. An integrated and personalized traveler information and incentive scheme for energy efficient mobility systems. Transportation Research Part C: Emerging Technologies, 2019.

[2] Wei Ma and Zhen Sean Qian. Estimating multi-year 24/7 origin-destination demand using high-granular multi-source traffic data. Transportation Research Part C: Emerging Technologies, 96:96–121, 2018.

[3] United States. Bureau of Public Roads. Traffic assignment manual for application with a large, highspeed computer, volume 37. US Department of Commerce, Bureau of Public Roads, Office of Planning, Urban Planning Division, 1964.

[4] PG Boulter and IS McCrae. Artemis: Assessment and reliability of transport emission models and inventory systems-final report. TRL Published Project Report, 2007.

Alternating Direction Method of Multipliers (ADMM) - Background

Solving linearly constrained optimization problems in form:

$$\min_{w,z} \quad h(w) + g(z) \qquad \text{s.t.} \quad Aw + Bz = c$$

Augmented Lagrangian function

$$\mathcal{L}(w, z, \lambda) \triangleq h(w) + g(z) + \langle \lambda, Aw + Bz - c \rangle + \frac{\rho}{2} \|Aw + Bz - c\|_2^2$$

Augmented update rules

$$\begin{aligned} &\text{Primal Update:} && w^{r+1} = \arg\min_{w} \mathcal{L}(w, z^r, \lambda^r), \\ & && z^{r+1} = \arg\min_{z} \mathcal{L}(w^{r+1}, z, \lambda^r) \\ &\text{Dual Update:} && \lambda^{r+1} = \lambda^r + \rho \left(Aw^{r+1} + Bz^{r+1} - c \right) \end{aligned}$$

Efficient Algorithm for Finding Optimal Incentives

$$\min_{\mathbf{S}} \sum_{\ell=1}^{|\mathcal{E}|}\sum_{t=1}^{|\mathbf{T}|} \hat{v}_{\ell,t} f_{CE}\left(\delta(\hat{v}_{\ell,t})\right) L_\ell$$

$$\text{s.t.}\quad \hat{v}_{\ell,t} = (\mathbf{RP})_{\ell,t}\mathbf{S1}$$

$$\mathbf{S}^T\mathbf{1} = \mathbf{1}$$

$$\mathbf{c}^T\mathbf{S1} \le \Omega$$

$$\mathbf{DS1} = \mathbf{q}$$

$$\mathbf{S} \in \{0,1\}^{(|\mathcal{R}||\mathcal{I}|)\times|\mathcal{N}|}$$

Relaxation & Reformulation

$$\min_{\boldsymbol{\gamma},\mathbf{u},\mathbf{S},\mathbf{W},\mathbf{H},\mathbf{z},\beta} \sum_{\ell=1}^{|\mathcal{E}|}\sum_{t=1}^{|\mathbf{T}|} \gamma_{\ell,t} f_{CE}\left(\delta(\gamma_{\ell,t})\right) L_\ell$$

$$\text{s.t.}\quad \mathbf{S1} = \mathbf{u},\quad \mathbf{W}^\intercal\mathbf{1} = \mathbf{1}$$

$$\mathbf{Du} = \mathbf{q},\quad \mathbf{Au} = \boldsymbol{\gamma}$$

$$\mathbf{H} = \mathbf{S},\quad \boldsymbol{\theta} = \mathbf{u}$$

$$\mathbf{c}^\intercal\boldsymbol{\theta} + \beta = \Omega,\quad \beta \ge 0$$

$$\mathbf{u} = \mathbf{z},\quad \mathbf{W} = \mathbf{S}$$

$$\mathbf{H} \in [0,1]^{(|\mathcal{R}|\cdot|\mathcal{I}|)\times|\mathcal{N}|}$$

ADMM

Algorithm 1 ADMM

Input: Initial values: $\boldsymbol{\gamma}^0$, $\mathbf{S}^0$, $\boldsymbol{\theta}^0$, $\mathbf{z}^0$, $\mathbf{H}^0$, $\mathbf{W}^0$, $\mathbf{u}^0$, β^0, $\lambda_1^0, \ldots, \lambda_9^0$, Dual update step: ρ, Number of iterations: T.
for $t = 0, 1, \ldots, T$ **do**
  for $\ell = 0, 1, \ldots, |\mathcal{E}|$ **do**
    for $\hat{t} = 0, 1, \ldots, |\mathbf{T}|$ **do**
      $\boldsymbol{\gamma}_{\ell,\hat{t}}^{t+1} = \underset{\boldsymbol{\gamma}_{\ell,\hat{t}}}{\operatorname{argmin}}\, \boldsymbol{\gamma}_{\ell,\hat{t}} f_{CE}\left(\delta(\boldsymbol{\gamma}_{\ell,\hat{t}})\right) L_\ell + \lambda_{4,(\ell,\hat{t})}^t(\mathbf{a}_{\ell,\hat{t}}\mathbf{u}^t - \boldsymbol{\gamma}_{\ell,\hat{t}}) + \frac{\rho}{2}(\mathbf{a}_{\ell,\hat{t}}\mathbf{u}^t - \boldsymbol{\gamma}_{\ell,\hat{t}})^2$
    end for
  end for
  $\mathbf{S}^{t+1} = \frac{1}{\rho}(-\lambda_1^t\mathbf{1}^\intercal + \lambda_5^t + \lambda_9^t + \rho\mathbf{u}^t\mathbf{1}^\intercal + \rho\mathbf{H}^t + \rho\mathbf{W}^t)(\mathbf{11}^\intercal + 2\mathbf{I})^{-1}$
  $\boldsymbol{\theta}^{t+1} = \frac{1}{\rho}(\mathbf{I} + \mathbf{c}^t\mathbf{c}^{t\intercal})^{-1}(-\lambda_6^t - \lambda_7^t\mathbf{c}^t + \rho\mathbf{u}^t - \rho\beta^t\mathbf{c}^t + \rho\Omega\mathbf{c}^t)$
  $\mathbf{z}^{t+1} = \frac{1}{\rho}(\lambda_8^t + \rho\mathbf{u}^t)$
  $\mathbf{H}^{t+1} = \Pi\left(\frac{1}{\rho}(-\lambda_5^t + \rho\mathbf{S}^{t+1})\right)_{[0,1]}$
  $\mathbf{W}^{t+1} = \frac{1}{\rho}(\mathbf{I} + \mathbf{11}^\intercal)^{-1}(-\mathbf{1}\lambda_2^{t\intercal} - \lambda_9^t + \rho\mathbf{11}^\intercal + \rho\mathbf{S}^{t+1})$
  $\mathbf{u}^{t+1} = \frac{1}{\rho}(3\mathbf{I} + \mathbf{D}^\intercal\mathbf{D} + \mathbf{A}^\intercal\mathbf{A})^{-1}(\lambda_1^t - \mathbf{D}^\intercal\lambda_3^t - \mathbf{A}^\intercal\lambda_4^t + \lambda_6^t - \lambda_8^t + \rho\mathbf{S}^{t+1}\mathbf{1} + \rho\mathbf{D}^\intercal\mathbf{q} + \rho\mathbf{A}^\intercal\boldsymbol{\gamma}^{t+1} + \rho\boldsymbol{\theta}^{t+1} + \rho\mathbf{z}^{t+1})$
  $\beta^{t+1} = \Pi\left(\frac{1}{\rho}(-\lambda_7^t - \rho\mathbf{c}^{t\intercal}\boldsymbol{\theta}^t + \rho\Omega)\right)_{\mathbb{R}_+}$
  $\lambda_1^{t+1} = \lambda_1^t + \rho(\mathbf{S}^{t+1}\mathbf{1} - \mathbf{u}^{t+1})$
  $\lambda_2^{t+1} = \lambda_2^t + \rho(\mathbf{W}^{t+1\intercal}\mathbf{1} - \mathbf{1})$
  $\lambda_3^{t+1} = \lambda_3^t + \rho(\mathbf{Du}^{t+1} - \mathbf{q})$
  $\lambda_4^{t+1} = \lambda_4^t + \rho(\mathbf{Au}^{t+1} - \boldsymbol{\gamma}^{t+1})$
  $\lambda_5^{t+1} = \lambda_5^t + \rho(\mathbf{H}^{t+1} - \mathbf{S}^{t+1})$
  $\lambda_6^{t+1} = \lambda_6^t + \rho(\boldsymbol{\theta}^{t+1} - \mathbf{u}^{t+1})$
  $\lambda_7^{t+1} = \lambda_7^t + \rho(\mathbf{c}^\intercal\boldsymbol{\theta}^{t+1} + \beta^{t+1} - \Omega)$
  $\lambda_8^{t+1} = \lambda_8^t + \rho(\mathbf{u}^{t+1} - \mathbf{z}^{t+1})$
  $\lambda_9^{t+1} = \lambda_9^t + \rho(\mathbf{W}^{t+1} - \mathbf{S}^{t+1})$
end for
Return: $\mathbf{S}^T$

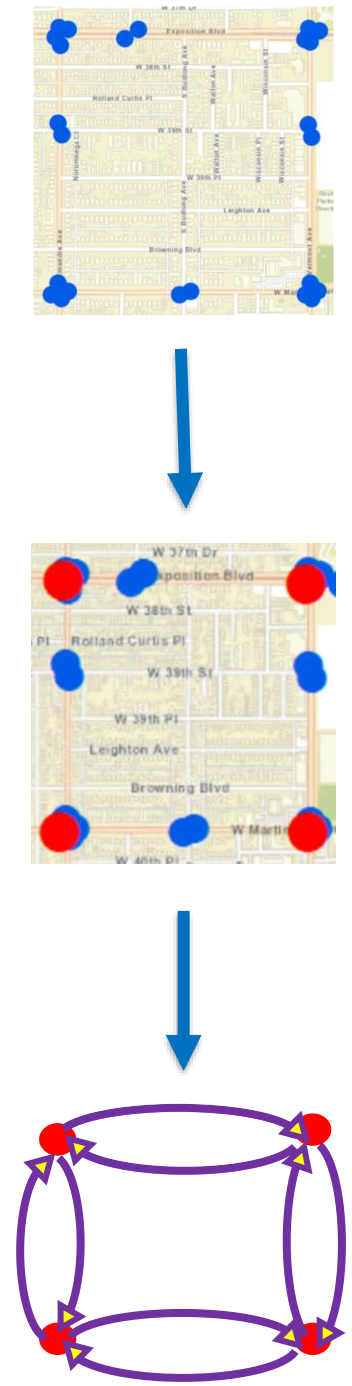
- The update rule of $\gamma_{\ell,t}$ can be done in parallel. Different columns of variables **W, S, H** can be updated in parallel (via edge computation).
- **Theorem:** The above algorithm finds an ϵ-solution of the relaxed problem in $O(1/\epsilon)$ iterations.
- How to do rounding? ADMM-Q algorithm (became popular recently for training binary neural networks)

Network Construction

❖ How do we construct the network?

❖ How to estimate O-D pairs for drivers?
- We do not have access to prior O-D as some works need [1-4]
- We have a large-scale problem (some prior work cannot scale)
- We use [5]

❖ Data:
- ADMS (Archived Data Management System at USC)
  - Real-time traffic data such as volume and speed
  - Collected by loop sensors
  - Highway data → recorded every 30 seconds
  - Arterial road data → recorded every 1 minute
- City: Los Angeles
- ➢ Why this region?
  1. Available detailed data
  2. Including both heavy and light traffic
- Date: March, April, and May 2018
- Only business days
- Used features: speed and volume



[1] P. Krishnakumari, H. v. Lint, T. Djukic, and O. Cats. A data driven method for od matrix estimation. Transportation Research Part C: Emerging Technologies, 2019.
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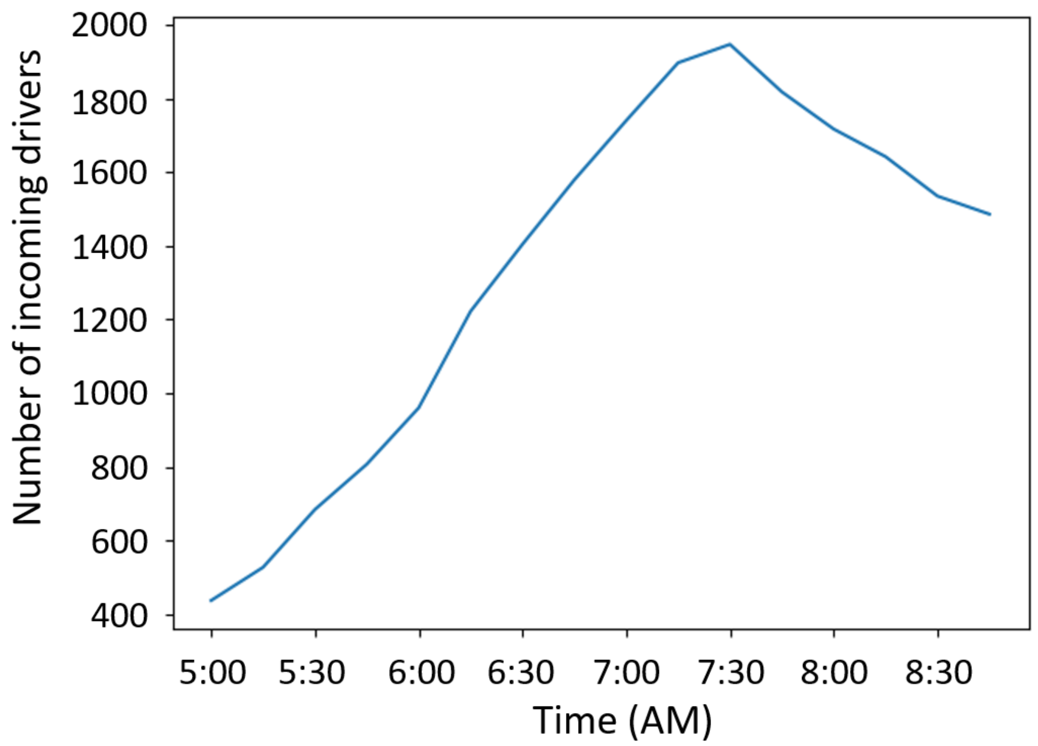
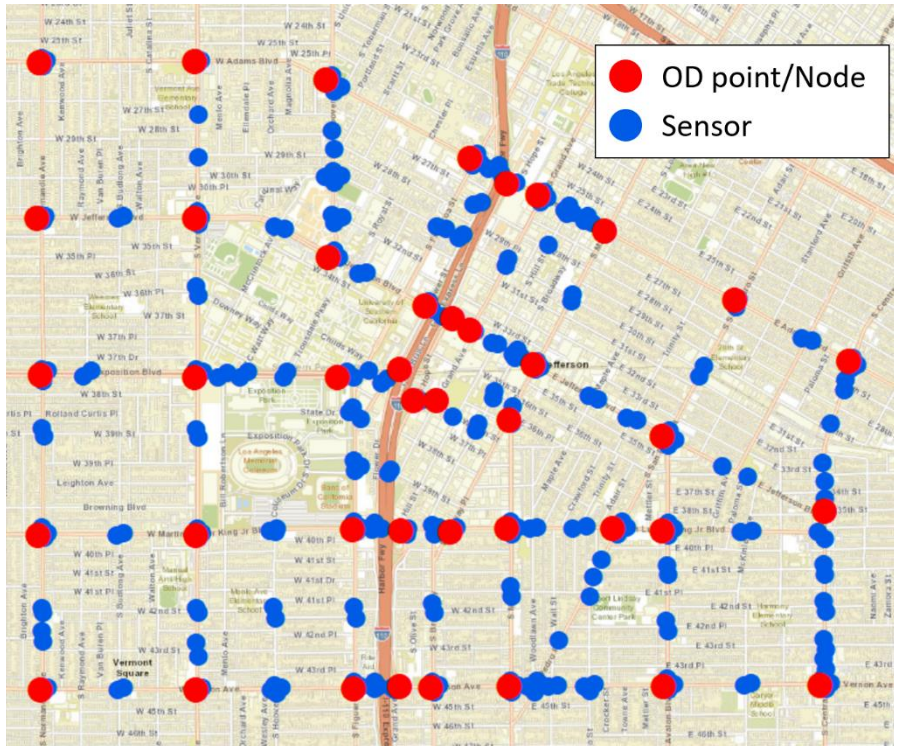
[5] W. Ma and Z. S. Qian. Estimating multi-year 24/7 origin-destination demand using high-granular multi-source traffic data. Transportation Research Part C: Emerging Technologies, 96:96–121, 2018

Numerical Experiments - Small Region

❖ Experiment I:
- Region: USC neighborhood
- Only arterial roads
- Incentive Set: {\$0, \$1, \$2, \$5, \$10, \$1000}

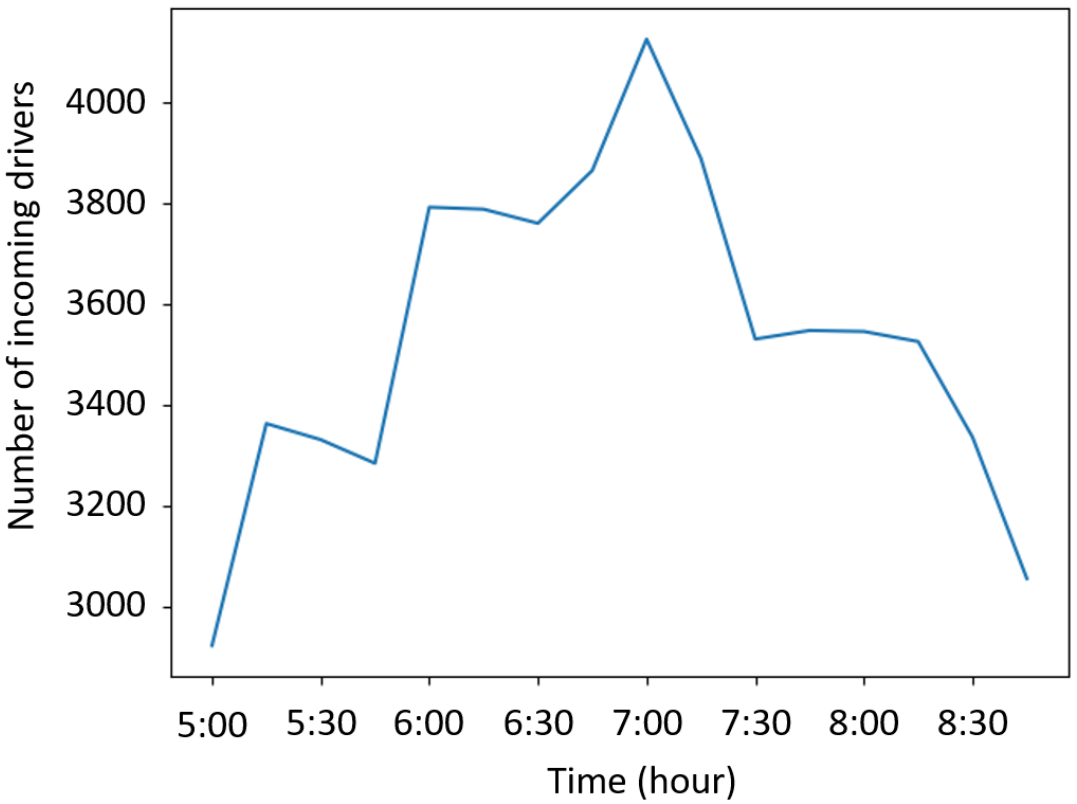




	Budget ($\$ \times 10^3$)	Percentage of drivers to whom we offered incentives	Average of the incentive amount (\$)	Reduction in Carbon Emission
7-8 AM exp. I	1	8.94%	1.51	4.33%
	10	43.12%	3.13	17.79%

Numerical Experiments - Large Region

- Experiment II:
  - Region: Los Angeles
  - Only highways
  - Incentive Set: {\$0, \$1, \$2, \$5, \$10, \$1000}



	Budget ($\$ \times 10^3$)	Percentage of drivers to whom we offered incentives	Average of the incentive amount (\$)	Reduction in Carbon Emission
7-8 AM exp. II	1	3.78%	1.75	0.72%
	10	21.91%	3.02	5.70%

Conclusion

- Offering personalized incentives to drivers to reduce congestion
- Efficient algorithms to solve the problem in large-scale
- Utilizing the computational power of individuals' smartphones by distributed algorithm

Download the code

❖ Future work:
  - Considering different travel modes such as public transportation, carpooling, and biking in options
  - Utilizing preference learning to learn the drivers' acceptance probability
  - More features such as income value and gender in computation the drivers' acceptance probability
  - Implementation and analysis of the algorithm in the real-world
  - Combining the data of highways and arterial ways

Thank you

Literature review

- Theory of congestion **pricing** has been widely studied (de Palma and Lindsey 2011, Tsekeris and Voß 2009)
  - Time or area dependent pricing (Zheng et al 2016)
  - Distance dependent (Daganzo and Lehe 2015)
  - Based on vehicle characteristics (Zhang et al 2018)

- Limitations:
  - Political barriers, social barriers such as equity, and unpopularity of taxation (Knockaert et al 2012, Levinson 2010, Martens et al 2012)

- Token-based schemes as an alternative idea (Verhoef et al 1997, Viegas 2001, Raux 2004).
  - Design and technological complexities (Azevedo et al 2018)

- **Offering rewards**
  - Psychologically more effective than penalizing (Brehm 1966)
  - More popular (Knockaert et al 2012)
  - Some studies on offering rewards:
    - Context of safe driving (Mazureck and Hattem 2006, Bolderdijk et al 2011)
    - Context of congestion reduction (Bliemer et al 2009, Knockaert et al 2012, Yue et al 2015)

A Simple Model

$$\min_{\text{incentives}} \quad \text{Cost of offering incentives}$$
$$\text{s.t.} \quad \mathbb{E}\left[\text{Volume}_t\right] \leq \text{Capacity}, \ \forall t$$

$$\min_{\{s_i^n\}} \quad \sum_{n \in \mathcal{N}} \sum_{i \in \mathcal{I}_n} s_i^n c_i^n$$
$$\text{s.t.} \quad \sum_{n \in \mathcal{N}} \sum_{i \in \mathcal{I}_n} \sum_{\mathbf{r} \in \mathcal{R}_n} s_i^n p_i^{\mathbf{r},n} \boldsymbol{\beta}_{\mathbf{r},t} \leq \mathbf{v}_0, \quad \forall t \in \mathbf{T}$$
$$\sum_{i \in \mathcal{I}_n} s_i^n = 1, \quad \forall n \in \mathcal{N},$$
$$s_i^n \in \{0, 1\}, \quad \forall n \in \mathcal{N}, \forall i \in \mathcal{I}_n$$

$\mathcal{N} = \{1, \ldots, N\}$	Set of drivers
$\mathcal{I}_n = \{(\text{money amount}, \text{route})\}$	Set of incentives for driver n
$s_i^n \in \{0,1\}, \quad i \in \mathcal{I}_n$	Decision variable: Offer incentive i or not
c_i^n	Cost of offering incentive i to driver n
$p_i^{r,n}$ [1]	Prob of selecting route $\boldsymbol{r}$ after offering incentive i
$\beta_{r,t}$ [2]	Location of driver on route $\boldsymbol{r}$ at time t (Probability vector)

[1] Chenfeng Xiong, Mehrdad Shahabi, Jun Zhao, Yafeng Yin, Xuesong Zhou, and Lei Zhang. An integrated and personalized traveler information and incentive scheme for energy efficient mobility systems. Transportation Research Part C: Emerging Technologies, 2019.

[2] Wei Ma and Zhen Sean Qian. Estimating multi-year 24/7 origin-destination demand using high-granular multi-source traffic data. Transportation Research Part C: Emerging Technologies, 96:96–121, 2018.

Modifying the Simple Model

$$\min_{\text{incentives}} \quad \text{Cost of offering incentives}$$
$$\text{s.t.} \quad \mathbb{E}\left[\text{Volume}_t\right] \leq \text{Capacity}, \ \forall t$$

↓

$$\max_{\text{incentives}} \quad U(\text{Drivers' travel time})$$
$$\text{s.t.} \quad \text{Volume}_t \leq \text{Capacity}, \ \forall t$$
$$\text{Cost of offering incentives} \leq \text{Budget}$$

↓ Sum utility (simple case)

$\mathcal{N} = \{1, \ldots, N\}$	Set of drivers
$\mathcal{I}_n = \{(\text{money amount}, \text{route})\}$	Set of incentives for driver n
$s_i^n \in \{0,1\}, \quad i \in \mathcal{I}_n$	Decision variable: Offer incentive i or not
c_i^n	Cost of offering incentive i to driver n
$p_i^{\boldsymbol{r},n}$	Prob of selecting route $\boldsymbol{r}$ after offering incentive i
$\beta_{\boldsymbol{r},t}$	Location of driver on route $\boldsymbol{r}$ at time t (Probability vector)

$$\min_{\{s_i^n\}} \sum_{n\in\mathcal{N}} \sum_{i\in\mathcal{I}_n} s_i^n \sum_{\mathbf{r}\in\mathcal{R}_n} p_i^{\mathbf{r},n} \delta_{\mathbf{r}}^n$$

Expected travel time of driver n after offering incentive i

$$\text{s.t.} \sum_{n\in\mathcal{N}} \sum_{i\in\mathcal{I}_n} \sum_{\mathbf{r}\in\mathcal{R}_n} s_i^n p_i^{\mathbf{r},n} \boldsymbol{\beta}_{\mathbf{r},t} \leq \mathbf{v}_0, \quad \forall t \in \mathbf{T}$$
$$\sum_{n\in\mathcal{N}} \sum_{i\in\mathcal{I}_n} s_i^n c_i^n \leq \text{Budget} = \Omega$$
$$\sum_{i\in\mathcal{I}_n} s_i^n = 1, \quad \forall n \in \mathcal{N},$$
$$s_i^n \in \{0,1\}, \quad \forall n \in \mathcal{N}, \forall i \in \mathcal{I}_n$$

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