

Planning for local delivery using sidewalk robots

Comparing optimized mothership vans methods

Wednesday, May 25, 2022

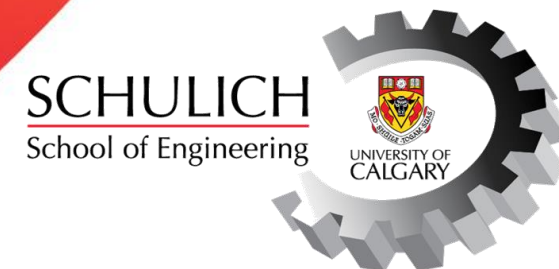
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Research Question and Agenda

1. Introduction

1. Past Research/Literature
2. Vehicle Characteristics

Research Question: What different ways can Sidewalk Robots be deployed from Motherships?

2. Methodology

1. Overview
2. Example
3. Analytical Results

Research Question: How can we estimate the travel distances on road and on sidewalks?

3. Default Design Case Study

1. Case Study Results
2. Sensitivity Analysis
3. Closed Form Results

Research Question: How does the proposed Mercedes-Benz design compare with a conventional truck?

4. Optimized Design

1. Design Variables
2. Changes and Impacts
3. Example

Research Question: Is the default design the cheapest way to implement the MS?

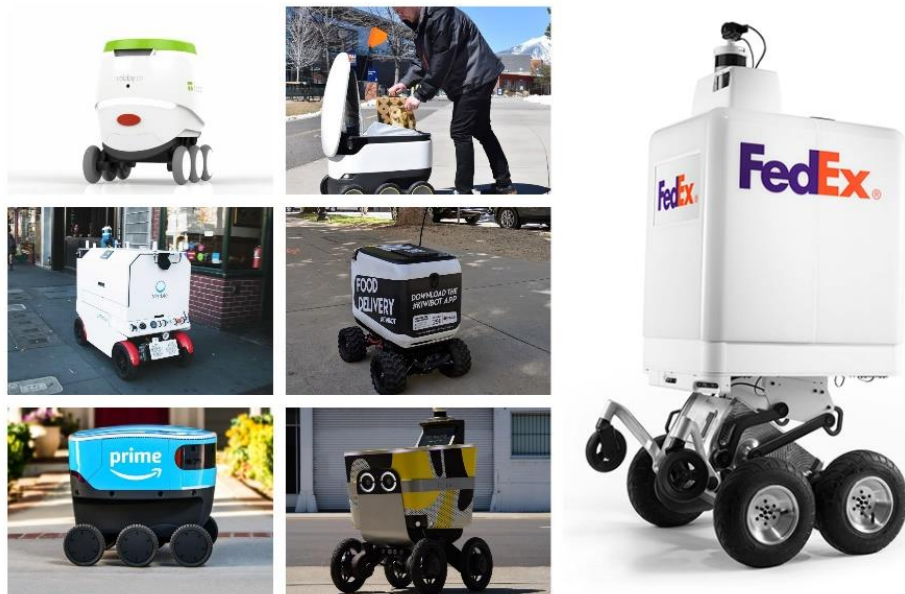
5. Conclusions



Introduction

Past Works, Terminology Contributions

Introduction – Technology and Terminology



SADR = Sidewalk Autonomous Delivery Robot

Vehicles of pedestrian scale, either fully autonomous or ‘human-in-the-loop’, that deliver light packages via a sidewalk network.

Also known as a “Person Delivery Device”.



MS = MotherShip Van

Vehicles capable of carrying one or more SADR plus additional packages for replenishment. Travels via the road network. May be autonomous or driven by a human.

Introduction – Proposal and Literature

Sept 2016: Mercedes-Benz and Starship Technologies released Mothership concept (*video in appendix*)

Reference	Classification of Strategy	MS Banned from Delivery	SADR Capacity	Methodology	Author's Problem Terminology
(Boysen et al., 2018)	MS Series	Yes	1	Mixed-Integer Program	Truck-based Robot Delivery (TBRD)
(Jennings & Figliozzi, 2019)	MS Series	Yes	1	Continuum Approximation	No terminology provided.
(Deng et al., 2020)	MS Tandem	No	1 - 25	Exact MIP and a Genetic Algorithm metaheuristic	Vehicle Routing Problem with Movement Synchronization (VRPMS)
(Simoni et al., 2020)	MS Tandem	No	1 - 3	Dynamic Program of Integer Program	Weighted Interval Scheduling Problem (WISP) of Traveling Salesman Problem with Robot (TSP-R)
(Yu et al., 2020)	MS Parallel	Yes	1 – 50	MILP, hybrid multi-start metaheuristic including destroy and repair operators together with a backtracking component	Two-Echelon Location Routing Problem (2E-LRP)
(Chen, Demir, & Huang, 2021)	MS Parallel	No	10kg	Adaptive Large Neighborhood Search heuristic algorithm	Vehicle Routing Problem with Time Windows and Delivery Robots (VRPTWDR)
(Chen, Demir, Huang, et al., 2021)	MS Parallel	No	1	Meta-heuristic of Mixed-Integer Linear Program	Vehicle Routing Problem with Time Windows and Delivery Robots (VRPTWDR)
(Ostermeier et al., 2022)	MS Parallel	Yes	1	Computational Heuristics and Algorithms	No terminology provided.
(Yu et al., 2022)	MS Tandem and MS Parallel	No	1 – 50	MILP solved with an adaptive large neighborhood search algorithm	Two-Echelon, Van-based Robot Hybrid Pickup and Deliveries (2E-VRHPD); Parallel Van and Robot Scheduling Problem with Hybrid Pickup and Delivery operations (PVRSP-HPD); a Two-Echelon Vehicle Routing Problem with Hybrid Pickup and Delivery operations (2E-VRP-HPD)

Introduction – Strategy Terminology

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N. Boysen et al./European Journal of Operational Research 271 (2018) 1085–1099

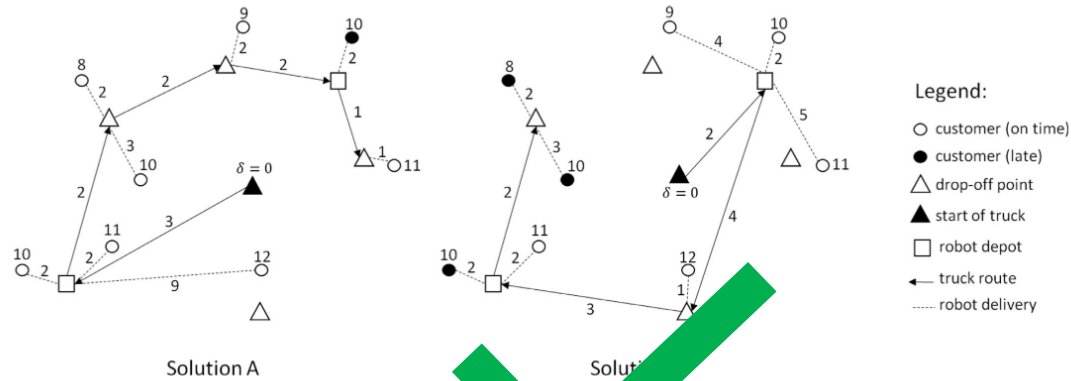


Fig. 2. Two alternative solutions for an example instance of TBRD.

MS Series
(Boysen et al., 2018)

Name describes the 'order' that the SADR's are deployed in.

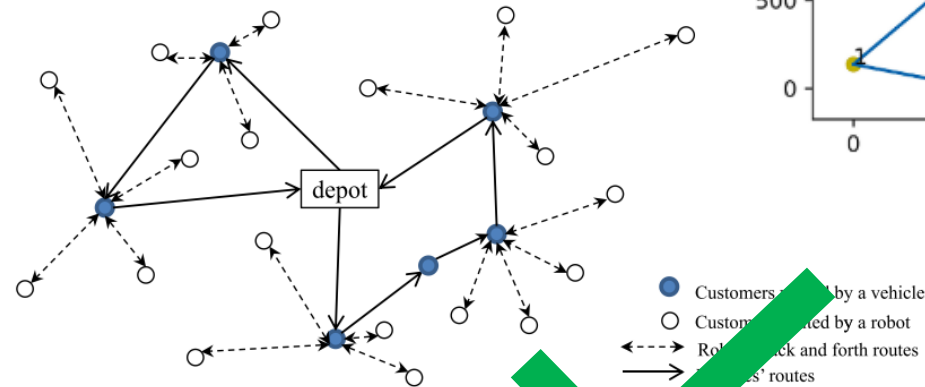
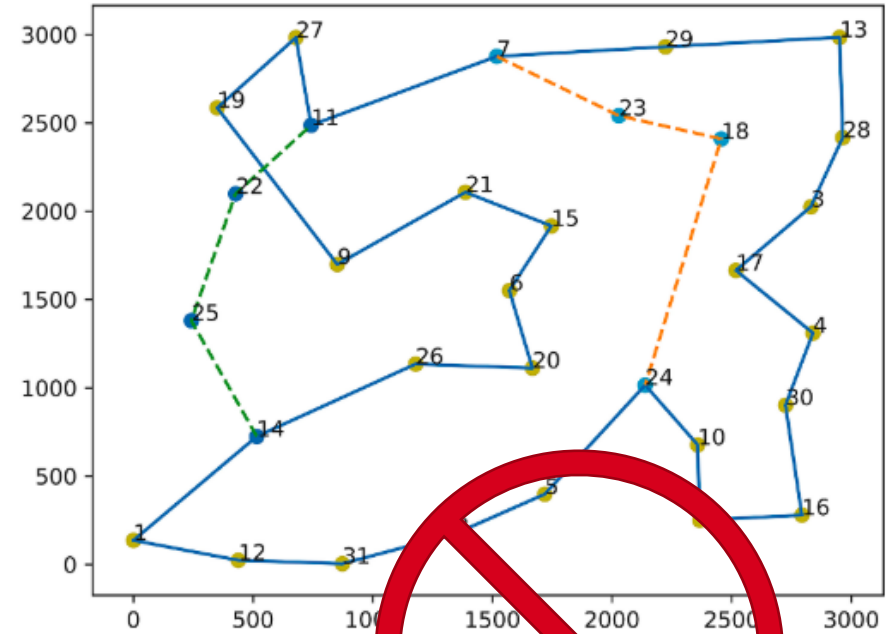


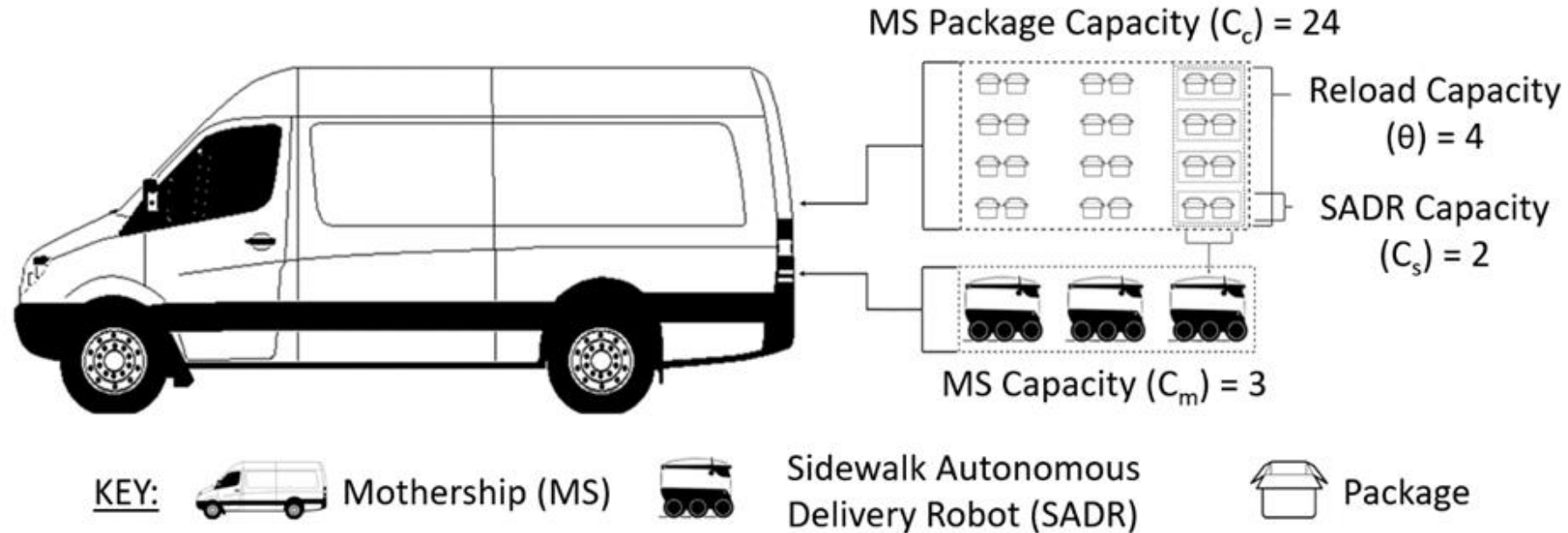
Fig. 2. An illustration of a feasible solution with 25 customers, 2 vehicles and 2 robots installed in each vehicle.

MS Parallel
(Chen et al., 2021)



~~**MS Tandem**
(Simoni et al., 2020)~~

Introduction – Vehicle Characteristics

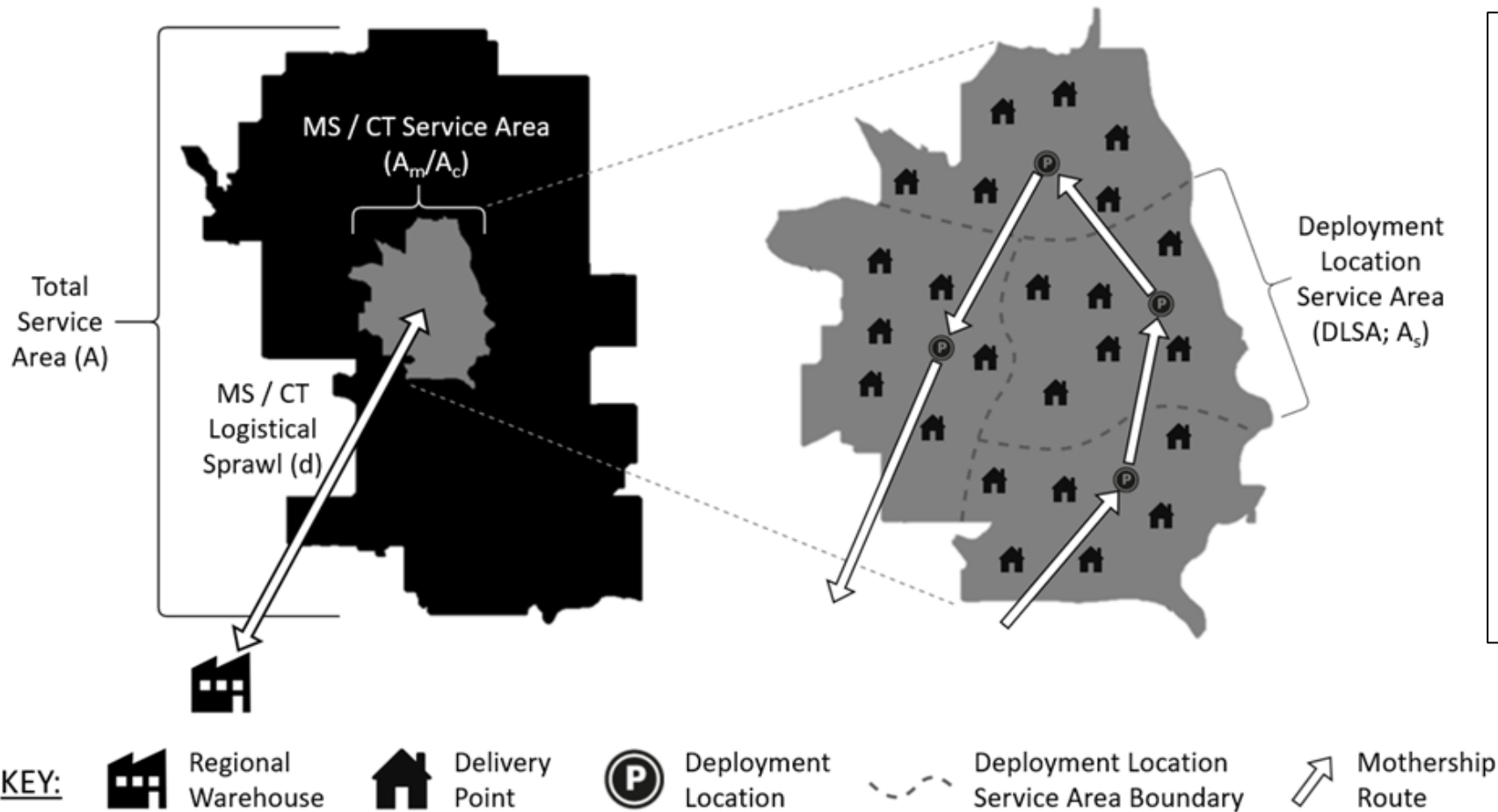


MS Capital Cost (α_s): \$222 per day
MS Transport Cost (β_s): 17¢ per kilometer
Assumed Gasoline MS Van

SADR Capital Cost (α_s): \$3.52 per day
SADR Transport Cost (β_s): 1.2¢ per kilometer
Assumed Electric SADR

Assumptions: Cost is modelled via travel distance; We do not consider vehicle speeds.
Vehicles always used to full capacity. Capacities equal between similar vehicles.
Operator may be a 3PL or company fleet.
Routes pre-planned at regional warehouse, and routes are reliable (deterministic).

Introduction – Regional Terminology



Assumptions:

- Uniform demand density (λ)*
- Uniform touring constant ($k = 0.87$)*
- Assumed Euclidean paths*
- Sufficient MS fleet size (m)*
- Sufficient SADR fleet size (s)*
- Sufficient deployment locations (P)*
- Sufficient time to conduct deliveries.*



Methodology

Overview, Example Application,
“Analytical Rules of Thumb” Table Summary

- Goal: Determine analytical expressions for on-road and on-sidewalk travel distance for each system (MS Series, MS Parallel, Conventional Truck [CT]).
- Method: Apply the following equations* and adapt as necessary.

For one vehicle in a multi-vehicle routing problem, the tour distance estimate is:

$$l(c, n, a, d) = 2 \cdot d + \frac{k \cdot \sqrt{a} \cdot (c - 1)}{\sqrt{n}}$$

l = distance per vehicle

c = vehicle capacity

a = service area

k = touring constant

For the fleet of vehicles in a multi-vehicle routing problem, the tour distance estimate is:

$$l_t(c, n, a, d) = 2 \cdot \frac{n}{c} \cdot d + \frac{k \cdot \sqrt{n \cdot a} \cdot (c - 1)}{c}$$

l_t = distance for fleet

n = number of delivery points

d = logistical sprawl (add more)

Reminder: Vehicles always used to full capacity. Capacities equal between similar vehicles.

**Equations adapted from Daganzo (2005) and Figliozzi (2008),*

Methodology – Application Example MS Series

Distance per SADR (D_{S1})

$$I_t(c, n, a, d) = 2 \cdot \frac{n}{c} \cdot d + \frac{k \cdot \sqrt{n \cdot a} \cdot (c - 1)}{c}$$

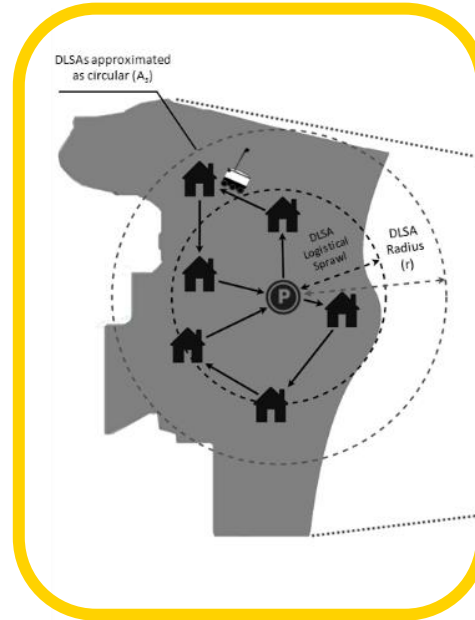
$$c = C_s$$

$$n = \theta \cdot C_s$$

$$a = A_{S1} = \frac{n}{\lambda} = \frac{\theta \cdot C_s}{\lambda}$$

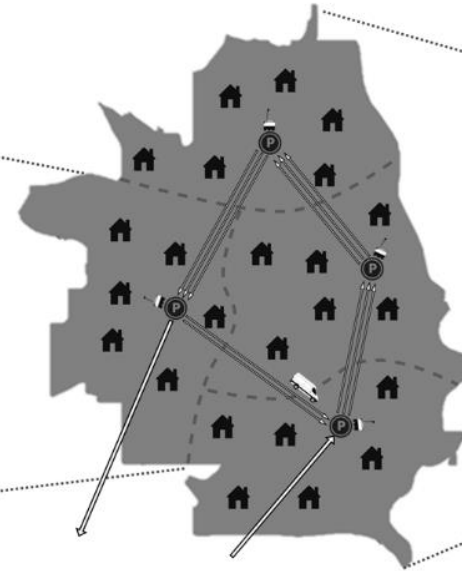
$$d = \frac{2 \cdot r}{3} = \frac{2 \cdot \sqrt{\theta \cdot C_s}}{3 \cdot \sqrt{\pi \cdot \lambda}}$$

$$D_{S1} = \frac{4 \cdot \sqrt{\theta^3 \cdot C_s}}{3 \cdot \sqrt{\pi \cdot \lambda}} + \frac{k \cdot \theta \cdot (C_s - 1)}{\sqrt{\lambda}}$$



(a) Per Deployment Location

n = sum of "H"
 d = two-thirds of DLSA Radius
 D_s = sum of "↗"



(b) Per MS Service Area

$n = \theta$ = sum of "P"
 d = logistical sprawl in "Total Service Area"
 D_M = sum of "↗"



(c) Total Service Area

m = sum of "↗"
 d = average of "↗"
Capacities in Example
 $C_s = 3; C_m = 4; \theta = 2$

KEY:



Regional Warehouse



Delivery Points



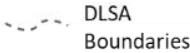
Deployment Locations



Motherships (MSs)



Sidewalk Autonomous Delivery Robots (SADRs)



DLSA Boundaries



Logistical Sprawls



MS Travel Distance

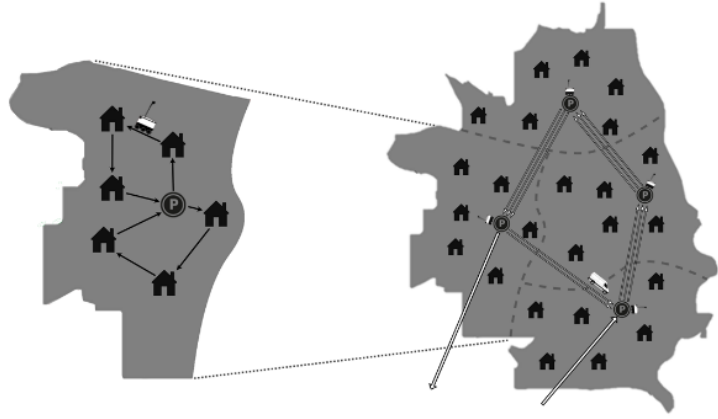


Logistical Sprawl of MS

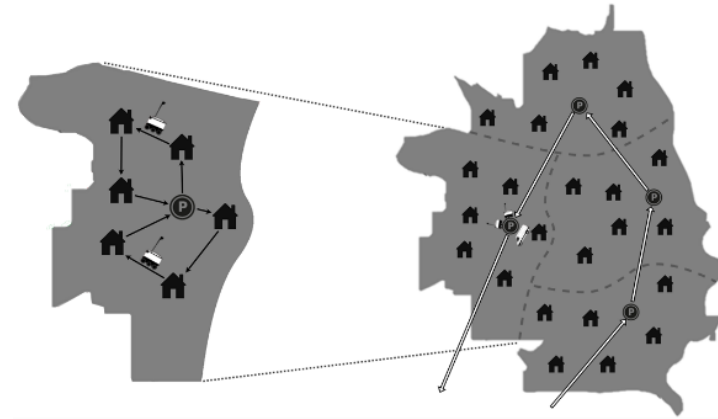


SADR Travel Distance

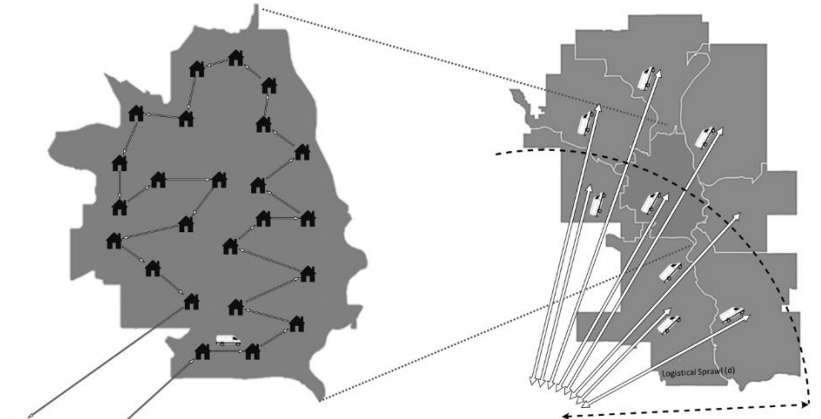
Methodology - Results



System 1: MS-Series



System 2: MS-Parallel



System 3: Conventional Truck



$$TD_{S1} = \frac{4 \cdot A \cdot \sqrt{\lambda} \cdot \theta}{3 \cdot \sqrt{\pi} \cdot C_s} + \frac{k \cdot A \cdot \sqrt{\lambda} \cdot (C_s - 1)}{C_s}$$

$$TD_{S2} = \frac{4 \cdot A \cdot \sqrt{\lambda} \cdot C_c}{3 \cdot C_s \sqrt{\pi} \cdot \theta} + \frac{k \cdot A \cdot \sqrt{\lambda} \cdot (C_s - 1)}{C_s}$$

$$TD_{S3} = 0$$



$$TD_{R1} = \frac{2 \cdot d \cdot A \cdot \lambda}{C_c} + \frac{(\theta + 1) \cdot k \cdot A \cdot \sqrt{\lambda}}{\sqrt{\theta} \cdot C_s}$$

$$TD_{R2} = \frac{2 \cdot d \cdot A \cdot \lambda}{C_c} + \frac{k \cdot A \cdot \sqrt{\lambda} \cdot (\theta - 1)}{\sqrt{C_c \cdot \theta}}$$

$$TD_{R3} = \frac{2 \cdot d \cdot A \cdot \lambda}{C_c} + \frac{k \cdot A \cdot \sqrt{\lambda} \cdot (C_c - 1)}{C_c}$$

C_s = SADR Capacity (#packages per SADR)
 C_m = MS Capacity (# SADR per MS)
 C_c = Package Capacity (#package per MS)

θ = Reload Capacity (Reloads of SADR per MS)
 λ = Demand Density (Packages per area)
 k = touring constant (0.87)

d = Logistical Sprawl (regional warehouse)
 R_s = Maximum range of SADR (per charge)

N.B. $C_c = C_s \cdot C_m \cdot \theta$

Analytical Comparison Summary

To minimize SADR distance use system in column compared to system in row

	MS-S	MS-P	CT
MS-S		When the MS capacity is greater than the reload capacity.	CT has no sidewalk distance
MS-P	When the reload capacity is greater than the MS capacity.		CT has no sidewalk distance
CT	CT has no sidewalk distance	CT has no sidewalk distance	

To minimize on-road (MS or CT) distance use system in column compared to system in row

	MS-S	MS-P	CT
MS-S		Equal distances when MS Capacity is equal to one.	When Reload Capacity is four less than SADR Capacity, at least three less.
MS-P	When MS Capacity is greater than one.		MS-P road distance always lower, or equal when SADR Capacity and MS Capacity equal one.
CT	When SADR Capacity is up to two greater than the Reload Capacity.	MS-P road distance always lower	

*Example of analytical comparison of MS Series vs MS Parallel strategies in slide appendix.
Please see upcoming publication for full explanation of each comparison.*

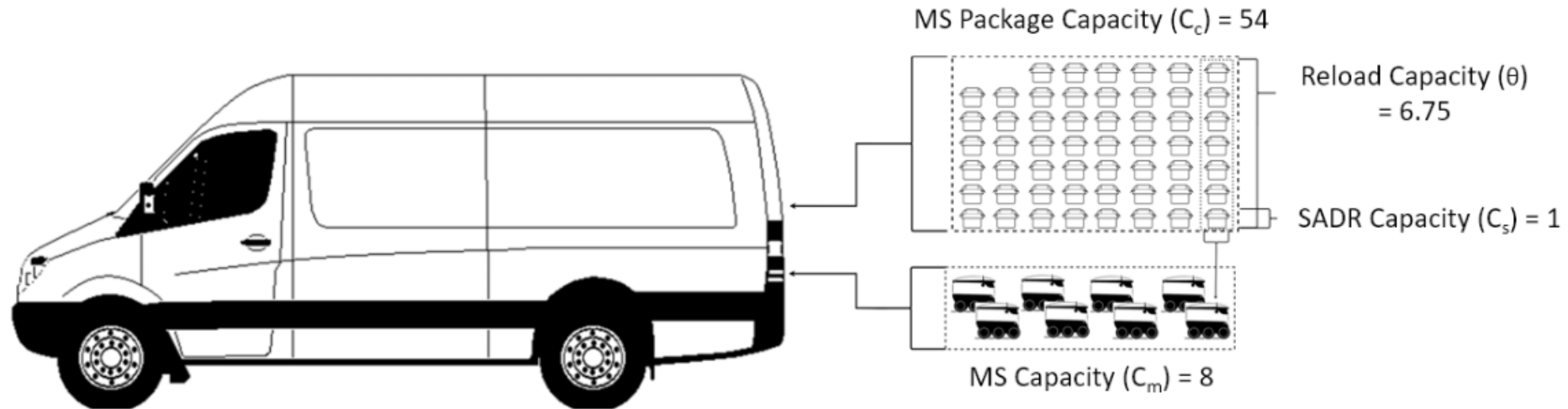


Default Design Case Study

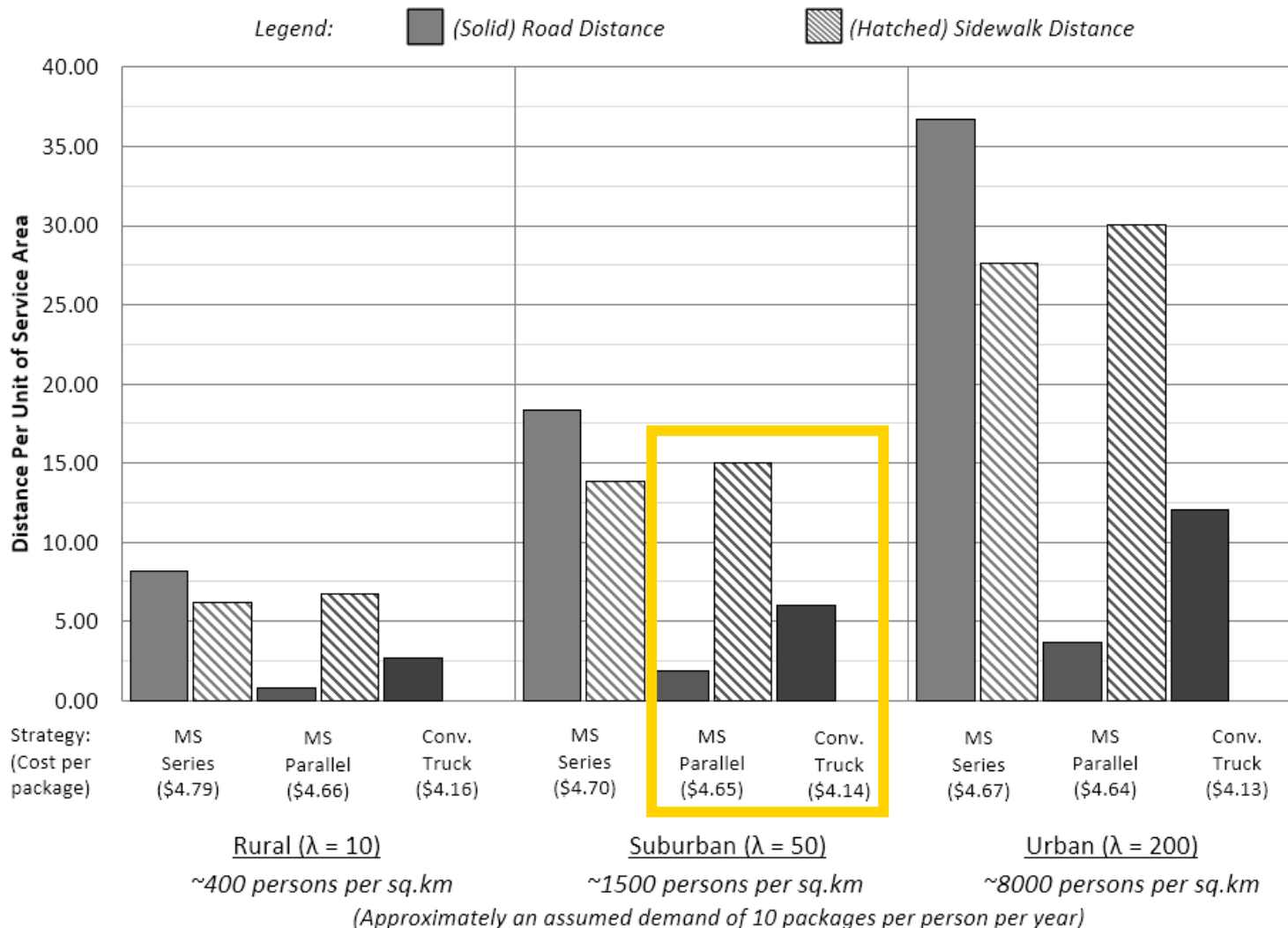
Evaluating the Mercedes Benz
Mothership and Starship
Technologies SADR



Default Design to Evaluate



Default Design Case Study Results



Vehicle Parameters

Model Mercedes Mothership and Starship Technologies SADR.

Item Capacity (C_c) = **54 packages**

SADR Capacity (C_s) = **1 package**

MS Capacity (C_m) = **8 SADRs**

Reload Capacity (θ) = **6.75 reloads**

Unit Costs:

CT Capital Cost (α_s):

\$222 per day, \$80,000 purchase

CT Transport Cost (β_s):

17¢ per kilometer, \$1.38/litre

MS Capital Cost (α_s):

\$222 per day, \$80,000 purchase

MS Transport Cost (β_s):

17¢ per kilometer, \$1.38/litre

SADR Capital Cost (α_s):

\$3.52 per day, \$2540 purchase

SADR Transport Cost (β_s):

1.2¢ per kilometer, 6¢ /kWh

$$\text{Total MS System Cost} = \text{Capital Cost per MS} * \text{Number of MS} + \text{MS Unit Transport Cost} * \text{MS Transport Distance} + \text{Capital Cost per SADR} * \text{Number of SADRs} + \text{SADR Unit Transport Cost} * \text{SADR Transport Distance}$$

$$\text{Total CT System Cost} = \text{Capital Cost per CT} * \text{Number of CT} + \text{CT Unit Transport Cost} * \text{CT Transport Distance}$$


Default Design Case Study Sensitivity Analysis


Default Case Study Experimental Results

	Default Value	Value for MS-Series TSC to equal CT TSC (% change)	Value for MS-Parallel TSC to equal CT TSC (% change)
Logistical Sprawl	0	N/A	N/A
Demand Density	50	N/A	0.021
SADR Transport Cost	\$0.0126	N/A	N/A
SADR Capital Cost	\$3.52	N/A	\$0.072 (-98%)
MS Transport Cost	\$0.1725	N/A	N/A
MS Capital Cost	\$222.22 (\$22.22 vehicle + \$200 labor)	\$191.57 (-14%)	\$194.63 (-12%)
CT Package Capacity	54	47 (-13%)	48 (-11%)

Analytical Closed Form Constraints

MS Series

 $\lambda > \left(\frac{4 \cdot \sqrt{C_c}}{3 \cdot R_s \cdot \sqrt{\pi} \cdot C_m} + \frac{k \cdot C_c \cdot (C_s - 1)}{R_s \cdot C_s \cdot C_m} \right)^2$

 $\lambda < \left(\frac{\beta_c \cdot k \cdot (C_c - 1) \cdot C_s \cdot C_m - \beta_m \cdot k \cdot \sqrt{C_m \cdot C_c^3} - \beta_m \cdot k \cdot C_s \cdot \sqrt{C_m^3 \cdot C_c} - \frac{4 \cdot \beta_s \cdot \sqrt{C_m \cdot C_c^3}}{3 \cdot \sqrt{\pi}} - \beta_s \cdot k \cdot C_c \cdot C_m \cdot (C_s - 1)}{\alpha_m \cdot C_s \cdot C_m + 2 \cdot \beta_m \cdot d \cdot C_s \cdot C_m + \alpha_s \cdot C_m^2 \cdot C_s - \alpha_c \cdot C_s \cdot C_m - 2 \cdot \beta_c \cdot d \cdot C_s \cdot C_m} \right)^2$

MS Parallel

$\lambda > \left(\frac{4 \cdot C_c}{3 \cdot R_s \cdot \sqrt{\pi} \cdot C_s \cdot C_m} + \frac{k \cdot C_c \cdot (C_s - 1)}{C_s \cdot C_m \cdot R_s} \right)^2$

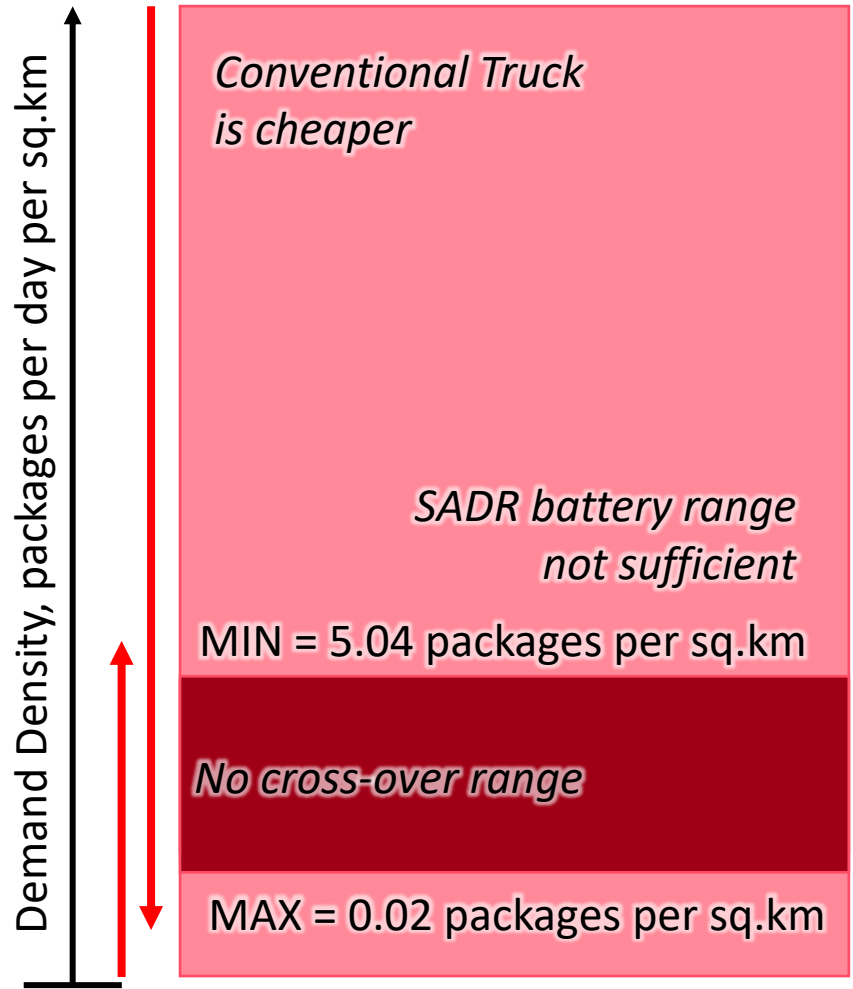
$\lambda < \left(\frac{(\beta_c \cdot k \cdot [C_c - 1] - \beta_m \cdot k \cdot [\theta - 1]) \cdot \sqrt{C_s \cdot C_m} - \beta_s \cdot C_c \cdot \left(\frac{4 \cdot \sqrt{C_m}}{3 \cdot \sqrt{\pi} \cdot C_s} + \frac{k \cdot (C_s - 1)}{C_s} \right)}{(\alpha_m - \alpha_c) + (\beta_m - \beta_c) \cdot 2 \cdot d + \alpha_s \cdot C_m} \right)^2$

Default Design Insights from Closed Form

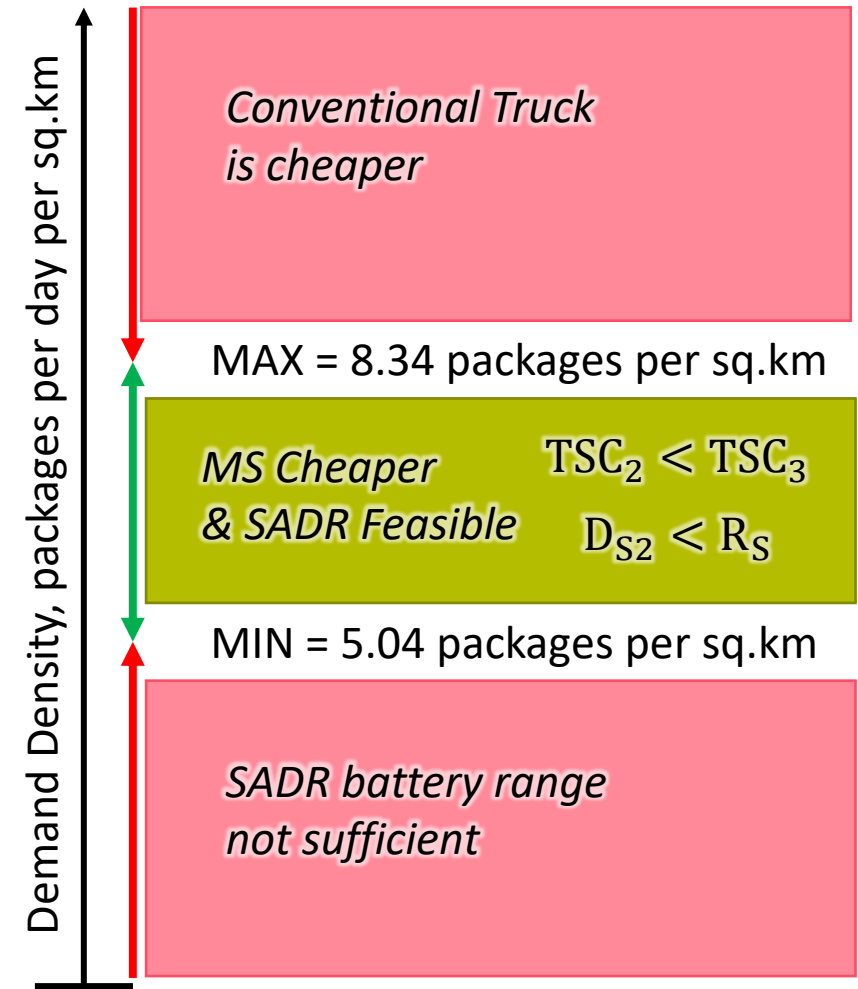
Vehicle Parameters
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 MS Transport Cost (β_s):
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 SADR Capital Cost (α_s):
 \$X per day
 SADR Transport Cost (β_s):
 1.2¢ per kilometer

SADR Capital Cost (α_s): \$3.52 per day



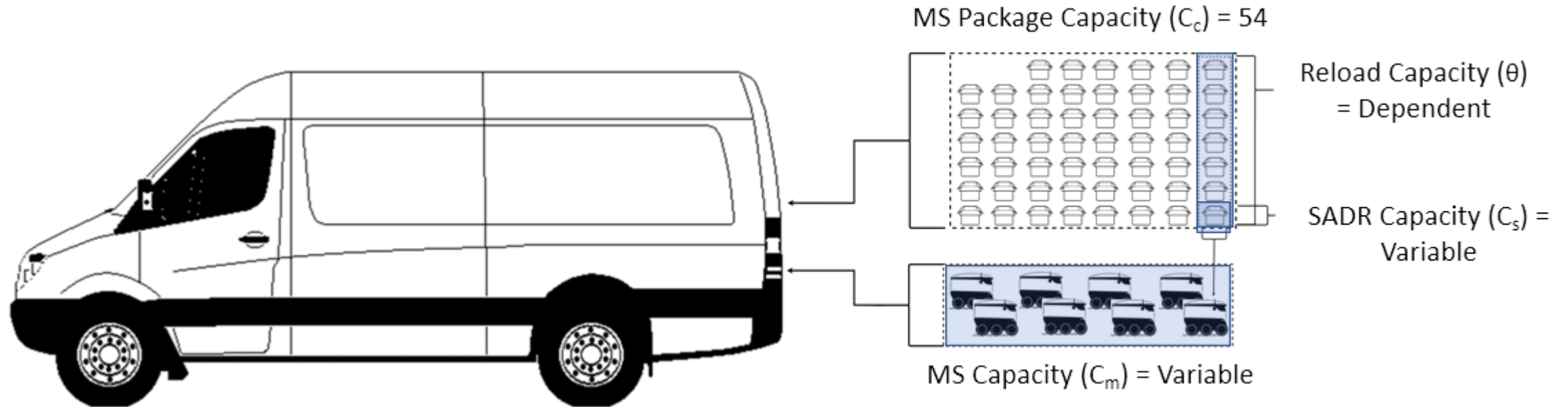
SADR Capital Cost (α_s): \$0.1725 per day



Optimized Design

Evaluating the Mercedes Benz Mothership and Starship Technologies SADR's

Optimized Design – Problem Definition



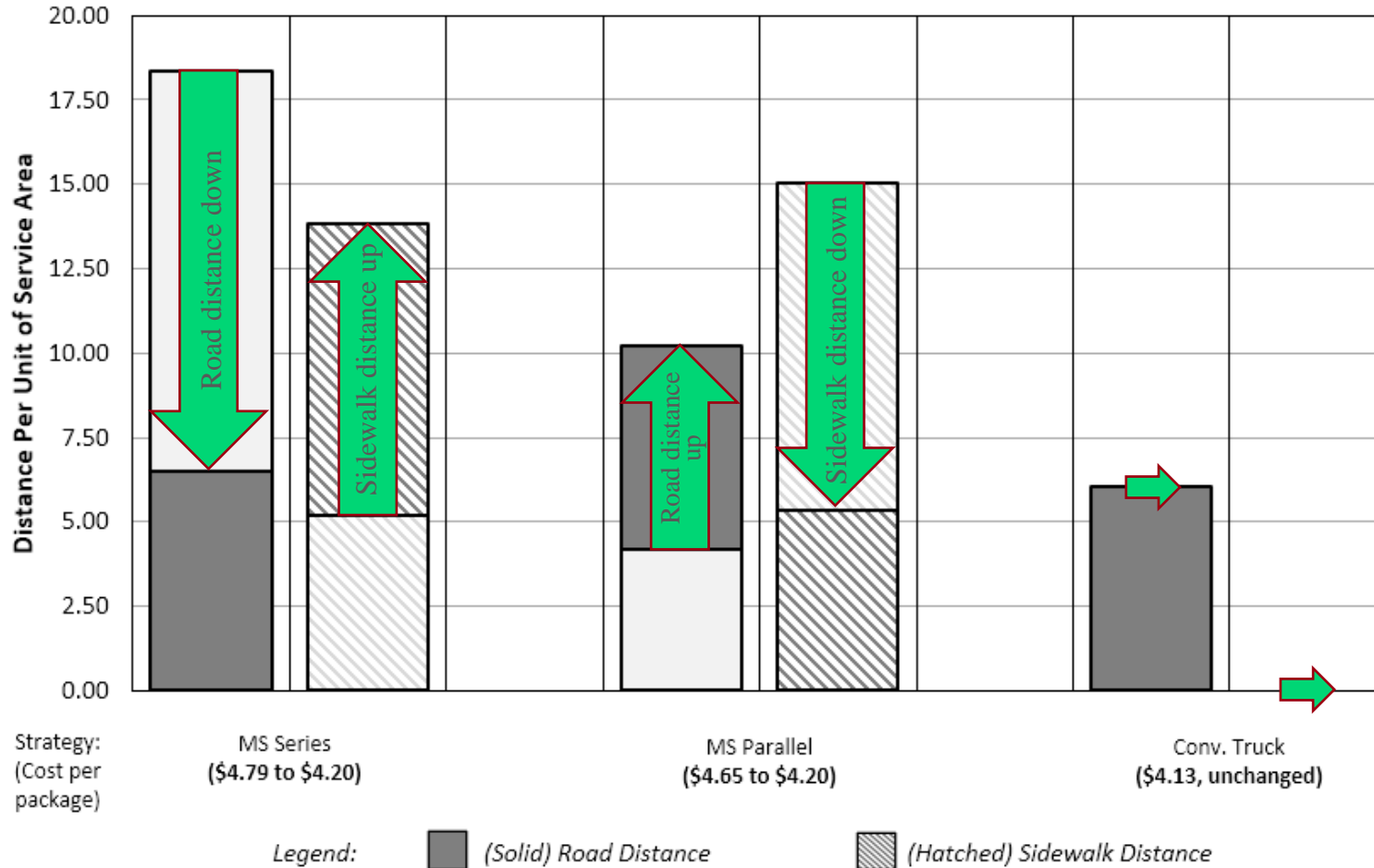
Objective: MINIMIZE Total System Cost (depends on MS Strategy).

Method: Integer Program Solver, Excel, Exhaustive Search

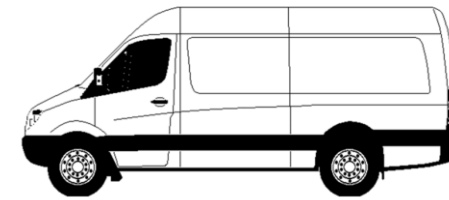
Subject to: SADR Range Constraint (depends on MS Strategy).
SADR Capacity, integer between 1 and 8
MS Capacity, integer between 1 and 8

Optimized Design – Changes and Impacts

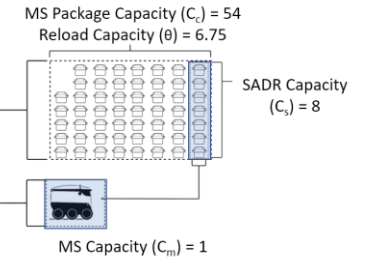
Suburban ($\lambda = 50$) ~1500 persons per sq.km and assumed demand of 10 packages per person per year



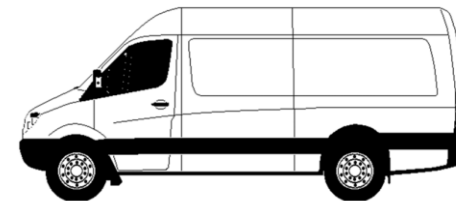
MS-Series



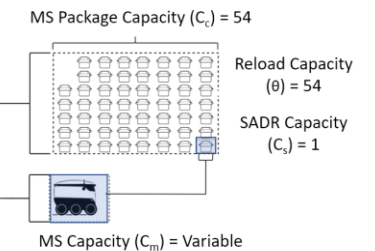
Fewer, larger SADR's



MS-Parallel



Fewer, small SADR's



Optimized Design – Example Varying Parameter

Lakewood Village Long Beach Once Monthly Deliveries (~100p/sq.km)

Downtown Long Beach Once Annually Deliveries (~25p/sq.km)

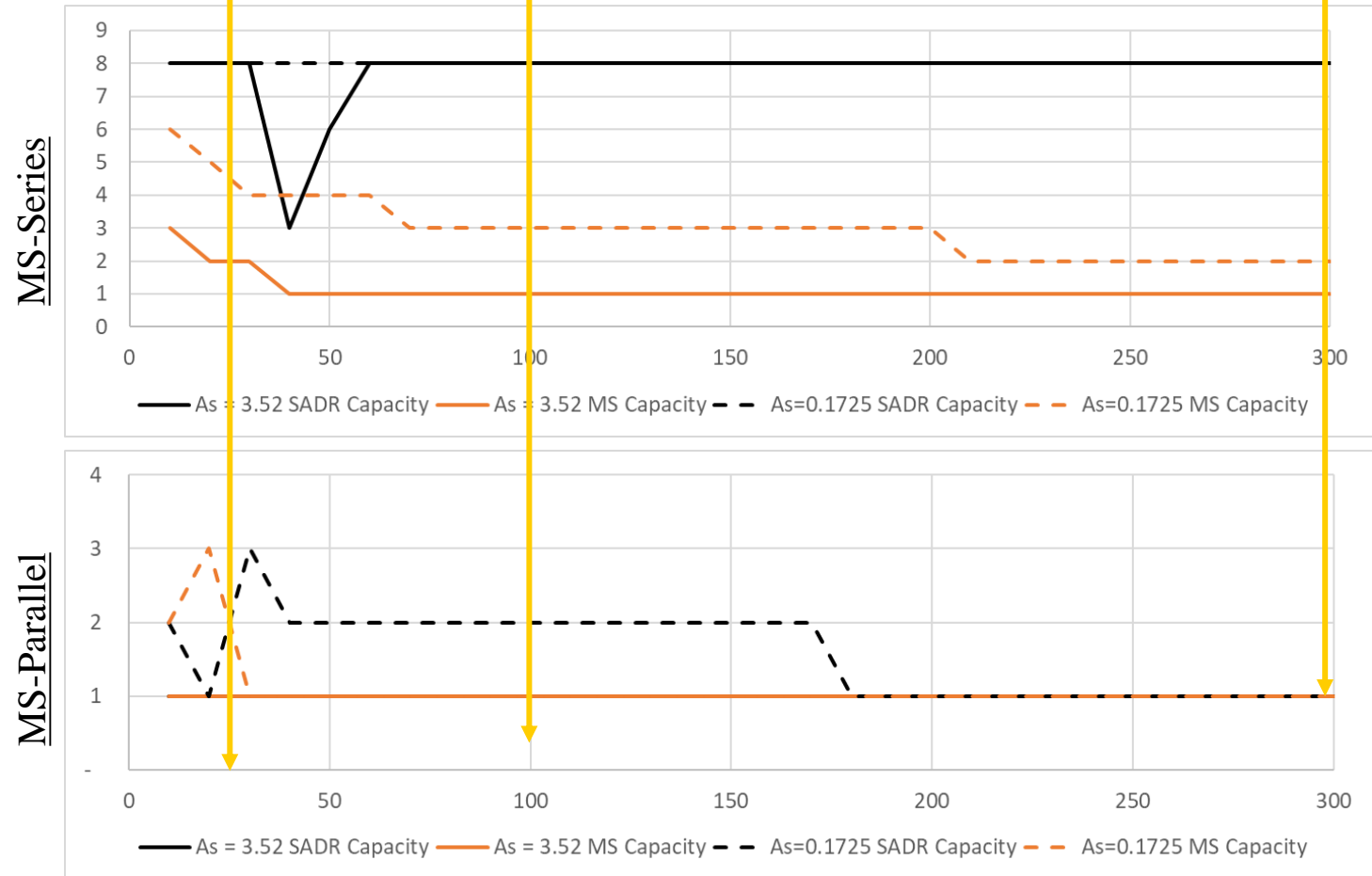
Downtown Long Beach Once Monthly Deliveries (~300p/sq.km)

MS-Series Insights

1. High demand density converges to stationary MS used as local hubs.
 1. Because of SADR Capital Costs
2. High SADR capacity preferred.
 1. Opposite to Mercedes design, with many SADRs with low capacity.

MS-Parallel Insights

1. Current SADR capital costs are too high to justify this strategy.
 1. Only used as our model enforces SADR use.
2. Lower SADR capital cost can mean a larger fleet is worthwhile.
 1. Customer time-window pressure is more likely to push and greater SADR fleet.



Thank you!

For more information or to submit further questions direct to me contact:

Email: Jacob.lamb@ucalgary.ca

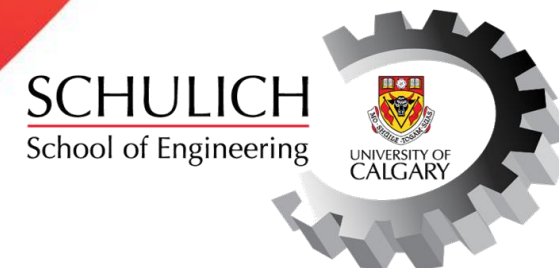
IISC: www.createiisc.com

Profile: [University Website](#)

or visit:



And await publication in review.



- Boysen, Nils, Stefan Fedtke, and Stefan Schwerdfeger. 2020. “Last-Mile Delivery Concepts: A Survey from an Operational Research Perspective.” *OR Spectrum* 43 (1). Springer Berlin Heidelberg: 1–58. doi:10.1007/s00291-020-00607-8.
- Chen, Cheng, Emrah Demir, and Yuan Huang. 2021. “An Adaptive Large Neighborhood Search Heuristic for the Vehicle Routing Problem with Time Windows and Delivery Robots.” *European Journal of Operational Research* 294 (3). Elsevier B.V.: 1164–80. doi:10.1016/j.ejor.2021.02.027.
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- Jennings, Dylan, and Miguel Figliozzi. 2019. “Study of Sidewalk Autonomous Delivery Robots and Their Potential Impacts on Freight Efficiency and Travel.” *Transportation Research Record* 2673 (6): 317–26. doi:10.1177/0361198119849398.
- Ostermeier, Manuel;, Andreas Heimfarth, and Alexander Hübner. 2022. “Networks - 2021 - Ostermeier - Cost-optimal Truck-and-robot Routing for Last-mile Delivery.Pdf.” *Networks* 79: 364–89.
- Simoni, Michele D., Erhan Kutanoglu, and Christian G. Claudel. 2020. “Optimization and Analysis of a Robot-Assisted Last Mile Delivery System.” *Transportation Research Part E: Logistics and Transportation Review* 142 (June). Elsevier: 102049. doi:10.1016/j.tre.2020.102049.
- Yu, Shaohua, Jakob Puchinger, and Shudong Sun. 2020. “Two-Echelon Urban Deliveries Using Autonomous Vehicles.” *Transportation Research Part E: Logistics and Transportation Review* 141 (June). Elsevier: 102018. doi:10.1016/j.tre.2020.102018.
- Yu, Shaohua, Jakob Puchinger, and Shudong Sun.. 2022. “Van-Based Robot Hybrid Pickup and Delivery Routing Problem.” *European Journal of Operational Research* 298 (3). Elsevier B.V.: 894–914. doi:10.1016/j.ejor.2021.06.009.

Appendix – Industry Video

VANS AND ROBOTS



Appendix – Problem

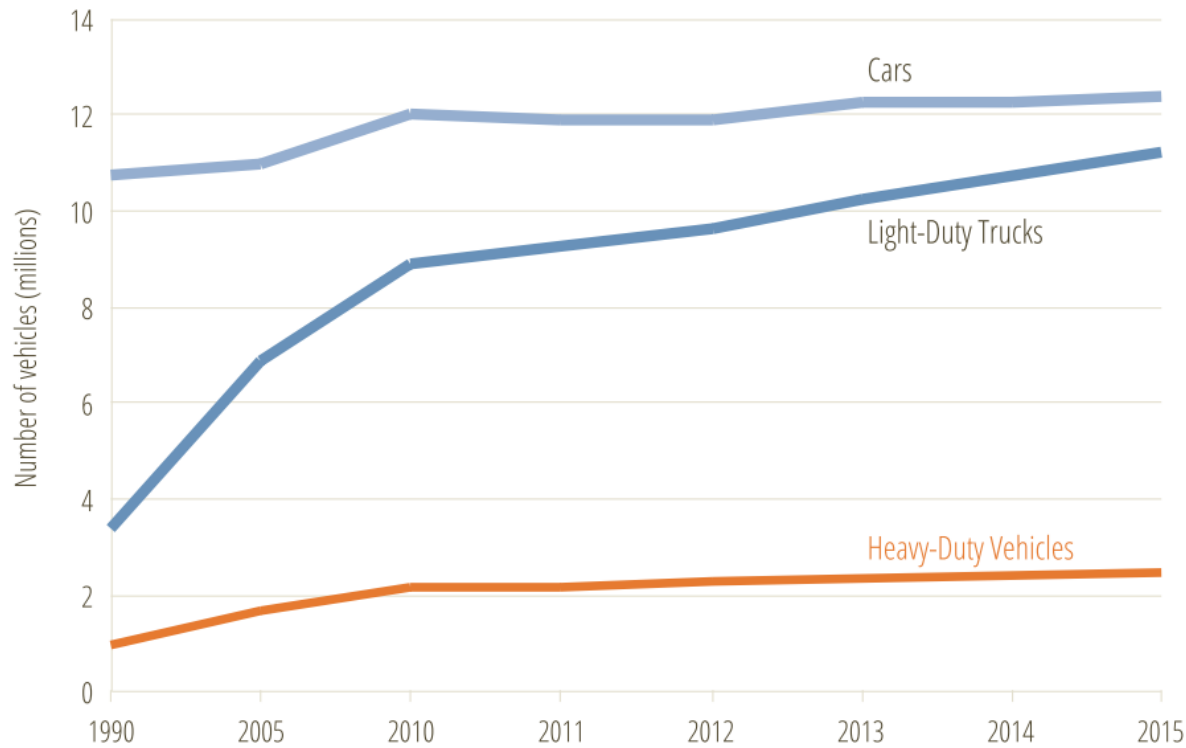


Figure 5. Trends in vehicle numbers in Canada

Data source: Environment and Climate Change Canada¹⁶

Bora, Plumtre, Eli Angen, and Dianne Zimmerman. 2017. "The State of Freight: Understanding Greenhouse Gas Emissions from Goods Movement in Canada." <https://www.pembina.org/reports/state-of-freight-report.pdf>.



US-based MSs: Starship Robots and Mercedes Benz (top), Digit by Ford (middle), ANYmal by ANYbotics (bottom)

Appendix – Analytical Comparison

When should you use MS Series for the least sidewalk travel?

$$C_c = C_m \cdot C_s \cdot \theta$$

When should you use MS Series for the least road travel?

$$TD_{S1} < TD_{S2}$$

$$\frac{4 \cdot A \cdot \sqrt{\lambda} \cdot \theta}{3 \cdot \sqrt{\pi} \cdot C_s} + \frac{k \cdot A \cdot \sqrt{\lambda} \cdot (C_s - 1)}{C_s} < \frac{4 \cdot A \cdot \sqrt{\lambda} \cdot C_c}{3 \cdot C_s \sqrt{\pi} \cdot \theta} + \frac{k \cdot A \cdot \sqrt{\lambda} \cdot (C_s - 1)}{C_s}$$

$$\frac{4 \cdot A \cdot \sqrt{\lambda} \cdot \theta}{3 \cdot \sqrt{\pi} \cdot C_s} < \frac{4 \cdot A \cdot \sqrt{\lambda} \cdot C_c}{3 \cdot C_s \sqrt{\pi} \cdot \theta}$$

$$\frac{\sqrt{\theta}}{\sqrt{C_s}} < \frac{\sqrt{C_c}}{C_s \sqrt{\theta}}$$

$$\theta < \frac{\sqrt{C_c}}{\sqrt{C_s}}$$

$$\theta < \frac{\sqrt{C_m \cdot C_m \cdot \theta}}{\sqrt{C_s}}$$

$$\theta < C_m$$

When there are more SADR per MS than reloads per MS.

When C_m equals 1.

$$\frac{2 \cdot d \cdot A \cdot \lambda}{C_c} < \frac{2 \cdot d \cdot A \cdot \lambda}{C_c} + \frac{k \cdot A \cdot \sqrt{\lambda} \cdot (\theta - 1)}{\sqrt{C_c \cdot \theta}}$$

$$\frac{2 \cdot d \cdot A \cdot \lambda}{\theta \cdot 1 \cdot C_s} < \frac{2 \cdot d \cdot A \cdot \lambda}{\theta \cdot 1 \cdot C_s} + \frac{k \cdot A \cdot \sqrt{\lambda} \cdot (\theta - 1)}{\theta \cdot \sqrt{1 \cdot C_s}}$$

$$0 < \frac{(\theta - 1)}{\theta}$$

$$1 < \theta$$

$$TD_{R1} < TD_{R2}$$

$$\frac{2 \cdot d \cdot A \cdot \lambda}{C_c} + \frac{(\theta + 1) \cdot k \cdot A \cdot \sqrt{\lambda}}{\sqrt{\theta} \cdot C_s} < \frac{2 \cdot d \cdot A \cdot \lambda}{C_c} + \frac{k \cdot A \cdot \sqrt{\lambda} \cdot (\theta - 1)}{\sqrt{C_c \cdot \theta}}$$

$$\frac{(\theta + 1) \cdot k \cdot A \cdot \sqrt{\lambda}}{\sqrt{\theta} \cdot C_s} < \frac{k \cdot A \cdot \sqrt{\lambda} \cdot (\theta - 1)}{\sqrt{C_c \cdot \theta}}$$

$$\frac{(\theta + 1)}{\sqrt{C_s}} < \frac{(\theta - 1)}{\sqrt{C_c}}$$

$$\frac{(\theta + 1)}{\sqrt{C_s}} < \frac{(\theta - 1)}{\sqrt{C_m \cdot C_s \cdot \theta}}$$

$$\sqrt{\theta} \cdot \frac{(\theta + 1)}{(\theta - 1)} < \frac{1}{\sqrt{C_m}}$$

$$\theta \cdot \left(\frac{\theta + 1}{\theta - 1} \right)^2 < \frac{1}{C_m}$$

Road travel is equal when there is only one SADR per MS.

Appendix – Total System Cost Estimates

Total MS System Cost = Capital Cost per MS * Number of MS + MS Unit Transport Cost * MS Transport Distance
+ Capital Cost per SADR * Number of SADRs + SADR Unit Transport Cost * SADR Transport Distance

$$TSC_{\#} = \alpha_m \cdot m + \beta_m \cdot TD_{R\#} + \alpha_s \cdot s + \beta_s \cdot TD_{S\#}$$

$$TSC_1 = \alpha_m \cdot \frac{A \cdot \lambda}{C_c} + \beta_m \cdot \left(\frac{2 \cdot d \cdot A \cdot \lambda}{C_c} + \frac{(\theta + 1) \cdot k \cdot A \cdot \sqrt{\lambda}}{\sqrt{\theta \cdot C_s}} \right) + \alpha_s \cdot \frac{A \cdot \lambda}{\theta \cdot C_s} + \beta_s \cdot \left(\frac{4 \cdot A \cdot \sqrt{\lambda} \cdot \theta}{3 \cdot \sqrt{\pi} \cdot C_s} + \frac{A \cdot k \cdot \sqrt{\lambda} \cdot (C_s - 1)}{C_s} \right)$$

$$TSC_2 = \alpha_m \cdot \frac{A \cdot \lambda}{C_c} + \beta_m \cdot \left(\frac{2 \cdot d \cdot A \cdot \lambda}{C_c} + \frac{k \cdot A \cdot \sqrt{\lambda} \cdot (\theta - 1)}{\sqrt{C_c \cdot \theta}} \right) + \alpha_s \cdot \frac{A \cdot \lambda}{\theta \cdot C_s} + \beta_s \cdot \left(\frac{4 \cdot A \cdot \sqrt{\lambda} \cdot C_c}{3 \cdot C_s \sqrt{\pi} \cdot \theta} + \frac{k \cdot A \cdot \sqrt{\lambda} \cdot (C_s - 1)}{C_s} \right)$$

Total CT System Cost = Capital Cost per CT * Number of CT + CT Unit Transport Cost * CT Transport Distance

$$TSC_3 = \alpha_c \cdot \frac{A \cdot \lambda}{C_c} + \beta_c \cdot \left(\frac{2 \cdot d \cdot A \cdot \lambda}{C_c} + \frac{k \cdot A \cdot \sqrt{\lambda} \cdot (C_c - 1)}{C_c} \right)$$

Appendix – Constraint Equations

MS Series



Range L.B. Constraint

\$Cheaper than CT U.B. Limit

$$\lambda > \left(\frac{4 \cdot \sqrt{C_c}}{3 \cdot R_s \cdot \sqrt{\pi \cdot C_m}} + \frac{k \cdot C_c \cdot (C_s - 1)}{R_s \cdot C_s \cdot C_m} \right)^2 \quad \lambda < \left(\frac{\beta_c \cdot k \cdot (C_c - 1) \cdot C_s \cdot C_m - \beta_m \cdot k \cdot \sqrt{C_m \cdot C_c^3} - \beta_m \cdot k \cdot C_s \cdot \sqrt{C_m^3 \cdot C_c} - \frac{4 \cdot \beta_s \cdot \sqrt{C_m \cdot C_c^3}}{3 \cdot \sqrt{\pi}} - \beta_s \cdot k \cdot C_c \cdot C_m \cdot (C_s - 1)}{\alpha_m \cdot C_s \cdot C_m + 2 \cdot \beta_m \cdot d \cdot C_s \cdot C_m + \alpha_s \cdot C_m^2 \cdot C_s - \alpha_c \cdot C_s \cdot C_m - 2 \cdot \beta_c \cdot d \cdot C_s \cdot C_m} \right)^2$$

MS Parallel



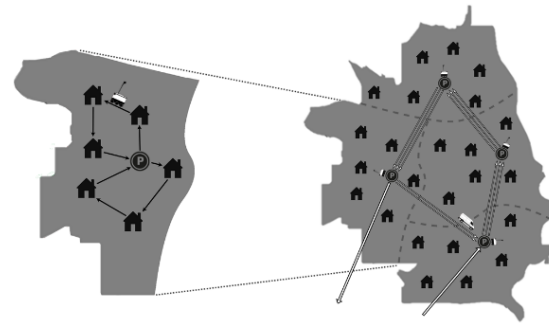
Range L.B. Constraint

\$Cheaper than CT U.B. Limit

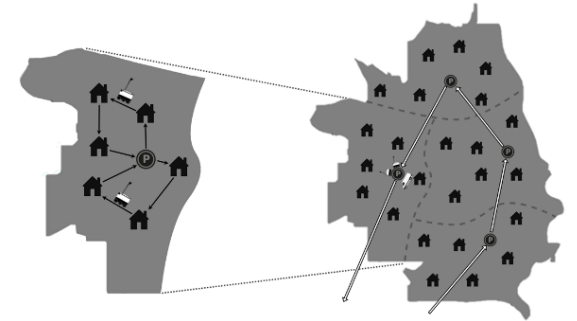
$$\lambda > \left(\frac{4 \cdot C_c}{3 \cdot R_s \cdot \sqrt{\pi \cdot C_s \cdot C_m}} + \frac{k \cdot C_c \cdot (C_s - 1)}{C_s \cdot C_m \cdot R_s} \right)^2 \quad \lambda < \left(\frac{(\beta_c \cdot k \cdot [C_c - 1] - \beta_m \cdot k \cdot [\theta - 1] \cdot \sqrt{C_s \cdot C_m}) - \beta_s \cdot C_c \cdot \left(\frac{4 \cdot \sqrt{C_m}}{3 \cdot \sqrt{\pi \cdot C_s}} + \frac{k \cdot (C_s - 1)}{C_s} \right)}{(\alpha_m - \alpha_c) + (\beta_m - \beta_c) \cdot 2 \cdot d + \alpha_s \cdot C_m} \right)^2$$

Appendix – Future Work

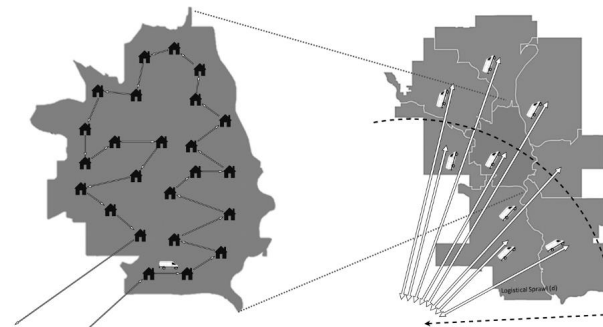
- Time Windows:
 - May be more appropriate to compare **two MS time windows against a longer CT tour.**
 - Further investigation of **time-window constraints in continuum approximation,** (Jennings and Figliozzi, 2019).
- Routing Approximation
 - Develop and validate **open vehicle routing approximations** so that Tandem SADR deployment systems may be modelled.
 - Develop and validate **different routing parameters (k)** more appropriate for small capacity vehicles and for small scale pathing, (Choi and Schonfeld, 2021)



System 1: MS-Series



System 2: MS-Parallel



System 3: Conventional Truck