Coordinated Freight Routing with Participation Incentives for Multiple Classes of Users

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Outline

- Motivation
- Previous Work
- Problem Formulation
- Mathematical Models
- Simulation Results
- Conclusion

Motivation

Motivation (1)

- In the **absence of cooperation**, the users of the transportation network act independently in an effort to minimize their own individual cost (e.g. travel time)
- This situation is known as **User Equilibrium (UE)** where no driver has an incentive to unilaterally change his/her route selection since he/she is not going to have a benefit from such a change
- The situation where the network users **cooperate** in a manner which contributes to the minimization of a “social cost” function (e.g. total travel time) is known as **System Optimum (SO)**
- The inefficiency of the UE compared to the SO in terms of the total social cost has been addressed to the literature as the **Price of Anarchy (PoA)**

Motivation (2)

- In **realistic transportation networks**, the **PoA** has been shown to reach the **value of 2 or even greater**, indicating the necessity for its reduction
- In a **SO solution**, some drivers may get harmed while some others may get benefit compared to the UE leading to **unfair** situations
- In **realistic transportation scenarios**, every driver has his/her own **Value-of-Time (VOT)**
- Due to **lack of cooperation**, truck drivers are not aware of the real-time demand for trips of the rest truck drivers

Motivation (3)

- **Question:** Does there exist a way to route the drivers in a manner where they do not get harmed compared to the UE leading to a fair situation while concurrently making the transportation network to approach as close as possible the SO solution?

- **Answer:** Use Game Theory and appropriate pricing schemes

Previous Work

Previous Work

- **Congestion Pricing:** Assign fees to each route to incentivize drivers to follow routes so that the network is driven to the SO solution
- **Uniform Revenue Refunding:** Share the money collected from congestion pricing uniformly among the drivers
- **Question:** What happens in the case of a non-deterministic scenario?
- **Question:** Can we make something more fair by pricing the truck drivers according to their Value of Time (VOT)?

Problem Formulation

Game Theoretic Formulation

- Use of a **non-atomic**, **symmetric information** game theoretic model assuming a **stochastic demand vector** for the truck drivers while the **coordinator knows the exact realization of the demand**
- The **truck drivers** are considered to be the "**players**" of the game and their objective is to **minimize** their **own individual travel time**
- **Coordinator** receives the Origin-Destination demands and additionally asks the truck drivers to declare their Value of Time (VOT), and then provides routing instructions by **minimizing** a "**social cost**" function

Mechanism Design Criteria

- Truck drivers should have a lower individual travel time compared to the UE in order to provide them **individual incentives for participation**
- Since the mechanism will compensate **truck drivers** who get harmed compared to the UE according to their declared VOT, guaranteeing that they **will truthfully declare their VOT** is of high importance in order to **avoid the exploitability of the mechanism**
- The resulting monetary scheme should be **budget balanced on average**
- Drive the network **as close as possible to the SO solution**

Mathematical Models

User Equilibrium (UE)

$$\begin{aligned} \underset{\alpha,\delta}{\text{minimize}} \quad & \lambda E[T_{tr}(\alpha)] + (1-\lambda)E[T_{tr}^{mon}(\alpha)] \\ \text{subject to} \quad & 0 \leq \alpha_{w,r}^{j} \perp F_{j,r}^{w}(\alpha) - \delta_{w}^{j} \geq 0, \ \forall j, w, r \\ & \sum_{r \in R_j} \alpha_{w,r}^{j} = 1, \ \forall j, w \end{aligned}$$

- **Minimize** a weighted combination of **total travel time** and **total monetary cost of trucks**

- **Due to lack of cooperation**, the truck drivers do not know the actual realization of the demand for the rest truck drivers

System Optimum (SO)

$$\underset{\alpha(\cdot)}{\text{minimize}} \quad \lambda E[T_s(\alpha)] + (1-\lambda)E[T_{tr}^{mon}(\alpha)]$$

$$\text{subject to} \quad \sum_{r \in R_j} \alpha_{w,r}^{c,j} = 1, \ \forall c, j, w$$

$$\alpha_{w,r}^{c,j} \geq 0, \ \forall c, j, w, r$$

- **Minimize** a weighted combination of the **total travel time of the network** and the **total monetary cost of trucks**

Congestion Pricing with Uniform Revenue Refunding

$$\underset{\alpha,\tau,\delta}{\text{minimize}} \quad \lambda E[T_s(\alpha)] + (1-\lambda)E[T_{tr}^{mon}(\alpha)]$$

$$\text{subject to} \quad 0 \leq \alpha_{w,r}^{j} \perp F_{w,r}^{j}(\alpha,\tau) - \delta_{w}^{j} \geq 0, \ \forall j, w, r$$

$$\sum_{r \in R_j} \alpha_{w,r}^{j} = 1, \ \forall j, w$$

$$\sum_{c} \sum_{j=1}^{v} \sum_{w=1}^{N} \sum_{r \in R_j} p_c d_{c,j}^{w} \alpha_{w,r}^{j} \tau_r^{j} = 0$$

- **Minimize** a weighted combination of the **total travel time of the network** and the **total monetary cost of trucks**

VOT Based Pricing

$$\underset{\alpha(\cdot),\tau(\cdot)}{\text{minimize}} \quad \lambda E[T_s(\alpha)] + (1-\lambda)E[T_{tr}^{mon}(\alpha)]$$

$$\text{subject to} \quad \sum_c \sum_{r \in R_j} p_c \alpha_{w,r}^{c,j} (J_{w,r}^{M,c,j} + \frac{1}{s_w} \tau_{w,r}^{c,j}) \leq \sum_c p_c A_{c,j}^{UE}, \ \forall j, w$$

$$\sum_c \sum_{r \in R_j} p_c \alpha_{i,r}^{c,j} (J_{i,r}^{M,c,j} + \frac{1}{s_i} \tau_{i,r}^{c,j}) \leq \sum_c \sum_{r \in R_j} p_c \alpha_{k,r}^{c,j} (J_{k,r}^{M,c,j} + \frac{1}{s_i} \tau_{k,r}^{c,j}), \ \forall j, i, k$$

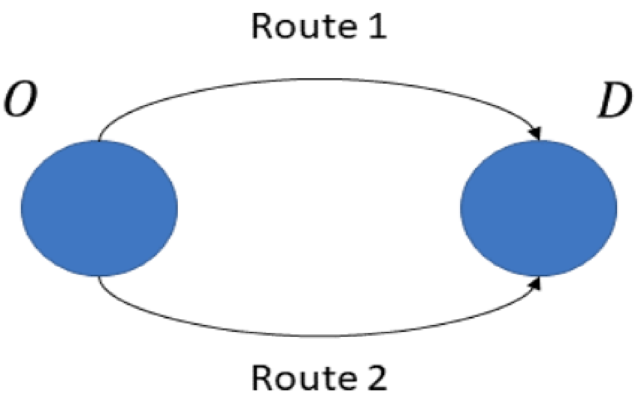
$$\sum_c \sum_{j=1}^{v} \sum_{w=1}^{N} \sum_{r \in R_j} p_c d_{c,j}^{w} \alpha_{w,r}^{c,j} \tau_{w,r}^{c,j} = 0$$

$$\sum_{r \in R_j} \alpha_{w,r}^{c,j} = 1, \ \forall c, j, w$$

$$\alpha_{w,r}^{c,j} \geq 0, \ \forall c, j, w, r$$

Simulation Results

Why not Congestion Pricing? (1)

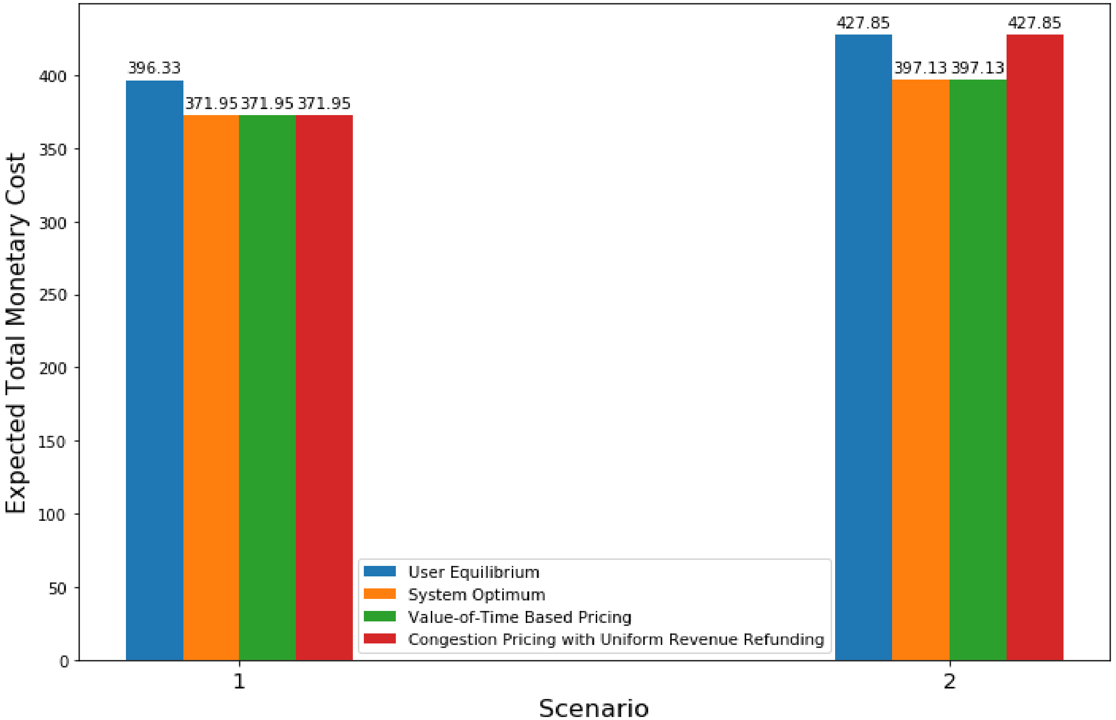


$$C_{1T} = 20 + X_{1T}^2$$

$$C_{2T} = 2 + 3X_{2T}^2$$

$$s_1 = 100\,\$/hr$$

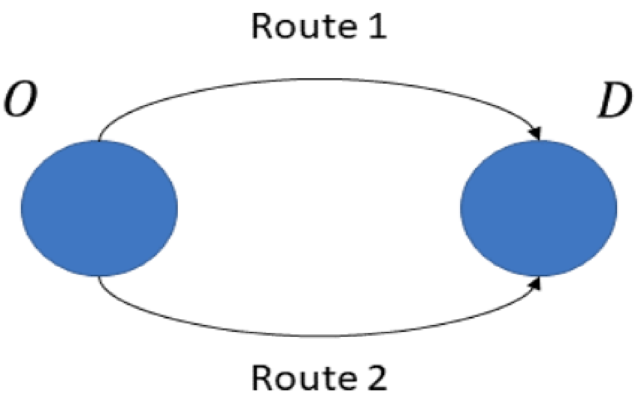
$$s_2 = 30\,\$/hr$$



Scenario 1: $d = [2 \quad 8]$ (deterministic)

Scenario 2: $d = [2 \quad 8] \quad w.p.\, 0.5$
$d = [3 \quad 7] \quad w.p.\, 0.5$

Why not Congestion Pricing? (2)

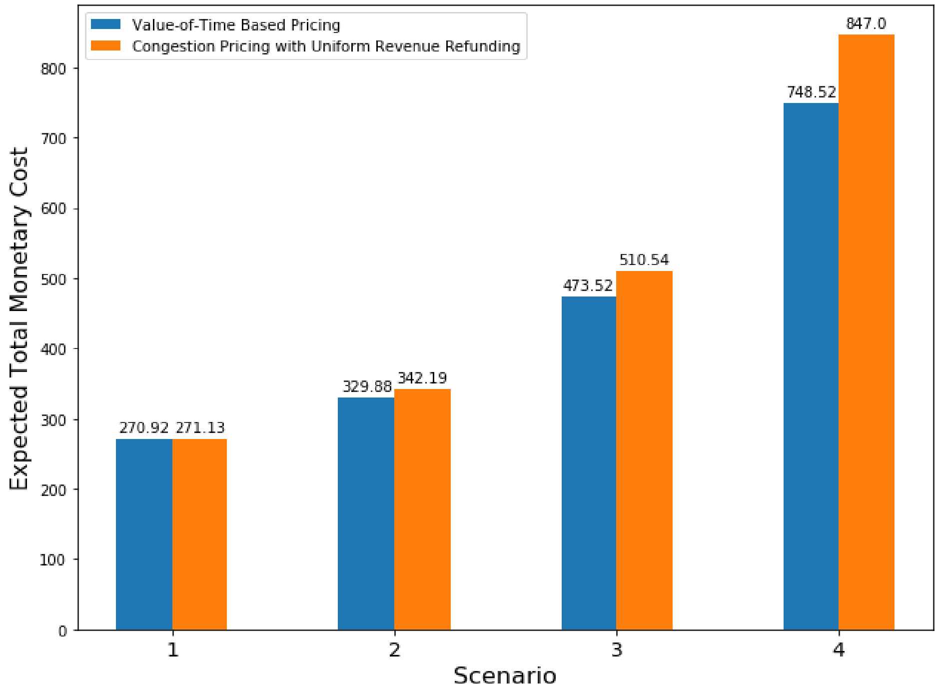


$$C_{1T} = 20 + X_{1T}^2$$

$$C_{2T} = 2 + 3X_{2T}^2$$

$$d(s_1) = \begin{cases} 2, & w.p.\ 0.5 \\ 5, & w.p.\ 0.5 \end{cases}$$

$$d(s_2) = \begin{cases} 4, & w.p.\ 0.5 \\ 8, & w.p.\ 0.5 \end{cases}$$



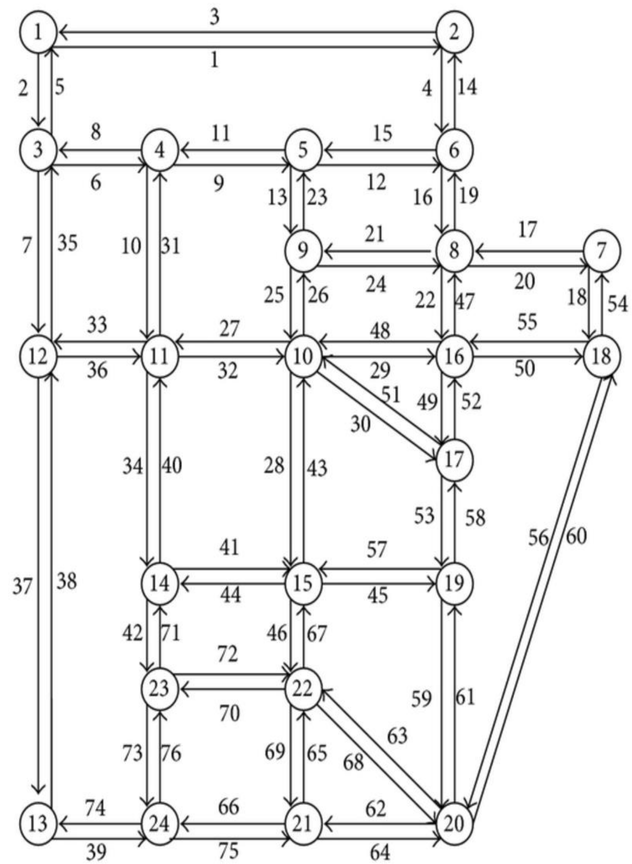
Scenario 1: $s_1 = 30\ \$/hr$, $s_2 = 30\ \$/hr$
Scenario 2: $s_1 = 50\ \$/hr$, $s_2 = 30\ \$/hr$
Scenario 3: $s_1 = 100\ \$/hr$, $s_2 = 30\ \$/hr$
Scenario 4: $s_1 = 200\ \$/hr$, $s_2 = 30\ \$/hr$

Sioux Falls Network (1)

- Application to the benchmark Sioux Falls network (24 nodes, 76 links)
- We assume that the truck drivers have only 6 available OD pairs: (1,7), (1,11), (10,11), (10,20), (15,5), (24,10)
- We have 2 classes of users with the following VOT:

$$s_1 = 200\ \$/hr$$

$$s_2 = 50\ \$/hr$$



Sioux Falls Network (2)

- The number of passenger vehicles at each link is considered to be constant

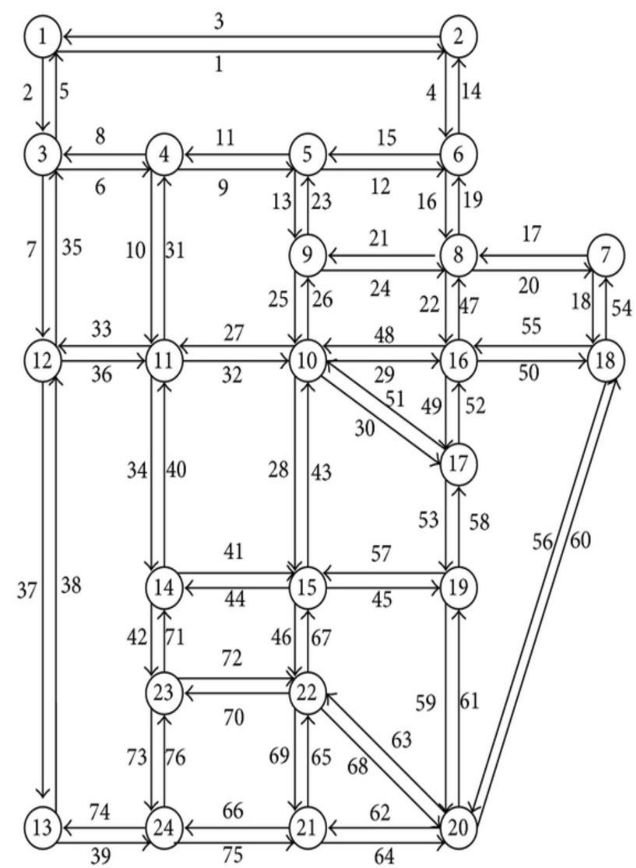
- The cost of each road segment is given by a BPR function (travel time):

$$C_{lp}^{t}(X_{lp}^{t}, X_{lT}^{t}) = C_{lT}^{t}(X_{lp}^{t}, X_{lT}^{t}) = \delta_a + \delta_b \left(\frac{X_{lp}^{t} + 3X_{lT}^{t}}{\delta_k} \right)^4$$

- The demand takes one of the following two equiprobable values:

$$d_1 = \begin{bmatrix} 3 & 4.5 & 6 & 3 & 14 & 3.6 \\ 1 & 2.8 & 5.4 & 7 & 9 & 2 \end{bmatrix}$$

$$d_2 = \begin{bmatrix} 5 & 1.8 & 3.9 & 15 & 6.4 & 2.4 \\ 6 & 5.5 & 1.8 & 6.5 & 11 & 6 \end{bmatrix}$$



Simulation Results

	UE	SO	CPURR	VOT-BP
$E[T_{tr}]$	53,574.4	49,082.5	53,570.3	49,049.8
$E[T_{tr}^{mon}]$	117,941.4	104,445.5	117,932.6	104,647.5
$E[T_s]$	167,160.7	157,924.8	167,152.0	157,934.9

- **VOT-BP** achieves 8.45% reduction in $E[T_{tr}]$, 11.27% reduction in $E[T_{tr}^{mon}]$ and 5.52% reduction in $E[T_s]$ while approaching the SO solution
- **CPURR** cannot "escape" the UE solution

Conclusion

- Congestion Pricing with Uniform Revenue Refunding (CPURR) cannot "escape" the UE solution in the the **absence of cooperation between the truck drivers**
- CPURR is inefficient when **different drivers have different VOT**
- We can ask the truck drivers to declare their VOT and **guarantee incentive compatibility**
- **Value-of-Time based Pricing** can **approach the SO solution** while concurrently **providing individual participation incentives**

End of Presentation

Questions ?